Conflict in the Shadow of Conflict

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We study how an advantage given to an interim winner in sequential conflicts characterizes dynamic competition between players and influences their payoffs. As the intensity of competition during each period is negatively correlated, perfect security is not necessarily desirable for contending parties. We present results which are widely applicable to various types of dynamic competition, where competition in each period is linked to the interim winner's relative advantage. Policy implications are also discussed in a variety of areas, and several extensions are explored.

Key Word: Sequential Conflicts, All-Pay Auctions,
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I. Introduction

We often observe that conflicts or competition among economic agents are not easily concluded. In particular, if a prize awarded after a contest is not instantly secured, the competing parties may have to endure a series of conflicts afterwards. For instance, even if a tribe assails another tribe and appropriates a valuable resource, the invaded tribe may not simply relinquish the resource but may attempt to retake it.[1] Similarly, although a firm may succeed in developing a new innovation and earning a patent, rivals may be able to imitate the innovation unless the patent is ironclad. As a result, they expect that a patent holder will end up in litigation against potential imitators. Cumulative innovations are also a type of sequential conflict. Suppose that the development of commercial technology would not be possible without the findings of basic research. When identical firms compete for research first and for development second, weak protection for basic research would trigger competition for the second innovation.

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[1] A large body of literature exists on conflict and appropriation, where property rights are not perfectly enforced and economic parties are contesting insecure properties. See the excellent survey paper by Garfinkel and Skaperdas (2006) on the detailed development of research in this field.

In these examples, the characteristics of the subsequent conflict are significantly different from those of the initial conflict. Once the winning party securely holds the prize, the successive contest is no longer a “fair” competition but rather a battle between protecting and stealing.[2] In such a case, it is likely that the defensive party has is in an advantageous position than the offensive party in most cases. To capture this aspect, we incorporate a measure of the degree of defensibility or security against potential threats. This can be thought of as the strength of a patent against an imitation, or the degree of forward protection of a patent in an environment of cumulative innovations.

More generally, this is the first winner’s relative advantage over her rival in the successive conflict, where she is favored in the next conflict given her higher winning probability. There are various types of dynamic competition in which the first winner has an advantage over a rival in subsequent contests. ICT industries, in which a large network effect prevails and/or switching costs exist, are good examples because an incumbent firm can enjoy a significant advantage in a subsequent contest. Moreover, considering sequential elections, a winner in an initial election often receives more media attention and financial support and will therefore have more opportunities to win in the next election. In these examples of dynamic competition, one can see that not only an immediate reward but also a relative advantage enjoyed by an initial winner subsequently can determine players’ effort or investment levels during the entire contest.

This paper initially studies how the advantage created in sequential conflicts characterizes the dynamic competition between players and influences their payoffs. We develop a two- period model of dynamic competition which is based on the literature of conflict economics, where players are involved in a battle in which the prize is not secured immediately. We then show that the results are generally applicable to dynamic competition, in which the competition at each stage is linked to the interim winner’s relative advantage in various aspects.

We start by demonstrating that the intensity levels of competition in the first period and in the second period are negatively correlated. If the property is perfectly defensible, agents fight only once in the first period. Not surprisingly, as the possibility of a second battle arises, agents play less aggressively in the first battle. In other words, as the property becomes less defensible, the intensity of competition is transmitted from the first period to the second. If it becomes perfectly non-defensible, they would fight only once, but in the second period.

Here, an interesting question is how the overall equilibrium competition, or more precisely the overall investment made by players, is characterized by the degree of defensibility. Although many types of dynamic competition have been studied, analyzing the overall competition over the long term from such a perspective has rarely been done. However, its importance cannot be disregarded, especially if one can manage dynamic competition to maximize contestants’ total efforts over the periods in question.

First, we first show non-monotonic relationships between the overall equilibrium

[2]Grossman and Kim (1995) make a distinction between offensive weapons and defensive fortifications. Their main focus was to show how the full security of claims to property can be achieved. In contrast, we attempt to explain how imperfect security influences intertemporal competition during sequential conflicts.

investments and the degree of defensibility. In doing this, there are two different cases. If marginal competition decreases in the magnitude of the prize, the equilibrium level of the investments overall takes on an inverse U-shape with regard to the degree of defensibility, with a unique maximum at a particular degree of insecurity. In contrast, if marginal competition increases, the equilibrium level of overall investments is then pseudo U-shaped with a unique minimum.

The result in the former case implies that the overall intensity of competition becomes stronger if the interim winner has a relative advantage in dynamic competition. More importantly, from the perspective of a competition planner, it becomes possible to maximize the expected effort levels by selecting the optimal relative advantage or disadvantage of the first winner; i.e., it becomes possible to award a favor, or impose disfavor, on the first winner during the second period, depending on the nature of the competition. When the first winner rarely has an advantage in subsequent contests, giving him a favor boosts dynamic competition. In contrast, when the first winner has too much of an advantage in successive contests, removing some advantages can increase players' effort levels in an environment of dynamic competition. For example, in repeated procurements, an incumbent firm which won in the first period may have cost advantages by learning by doing or transferable investments in the second period. In such a case, an auctioneer prefers to remove the incumbent's advantages by requiring the winner to share information or the outcome of investments with rivals.

We also extend the model in several ways to show the robustness of our results and explore several other interesting implications. First, the basic result will hold when the interim winner has an advantage in payoffs. Second, we analyze n-period contests after which we investigate how uncertainty regarding the degree of security affects the intensity of competition in both periods. Last, we relax the assumption of winner-take-all competition. We believe that all of these extensions yield worthwhile results.

In the literature on optimal contests or all-pay auctions, there are a number of papers on how a contest designer can increase the overall level of investments of contestants in sequential contests. Baye, Kovenock and de Vries (1993) showed that is occasionally better for the seller to exclude some buyers in order to increase her expected revenue. Clark and Riis (1998) explored whether to distribute prizes sequentially or simultaneously. Gradstein and Konrad (1999) compared simultaneous contests and a series of pairwise contests. Moldovanu and Sela (2001) studied how prizes should be allocated and whether there should be one prize or more than two prizes depending on the shapes of cost functions.[3] However, our paper addresses this issue from the perspective of a designer who attempts to control a relative (dis)advantage of a previous winner so as to elicit maximum efforts from contest participants.[4] We also examine how this capability of a

[3]Many recent papers study optimal prize allocation rules in dynamic contests using experiments, including Cason *et al.* (2010) and Sheremeta (2010) among others. See Dechenaux (2015) for a more detailed survey in this area.

[4]Meyer (1991, 1992), in a dynamic setting, drew conclusions similar to ours when arguing that an optimal contract should have a positive bias. With the more general setting provided here, however, it should be negatively biased when the first winner has a sufficiently large advantage. Similarly, several implications of the findings by Laffont and Tirole (1988) are similar to those here, as they show that an incumbent with transferable investments should be favored at the reprocurement stage.

designer is related to the various tactics available to him, including the division of rewards, extending the contest periods, and imposing different degrees of uncertainty on the security of the prize.

The paper is organized as follows. In the next section, we explain the conflict technology and the setup of a basic model. We characterize equilibrium and analyze the results and then discuss applications of the model and its policy implications in Sections III and IV. In Section V, we extend the model in several ways to show the robustness of the results and investigate additional interesting findings. Section 6 concludes the paper.

II. Basic Model

Consider the following two-period model. There are two risk-neutral agents. Agent 1 and 2 contest an exogenous prize, *R*. This prize can be thought of as a newly found diamond mine, an increase in profits from developing a new technology, or a license for a new business. First, they choose the weapons level and fight against each other for the prize. The battle is a winner-take-all contest. Given that g_i and g_j represent the quantity of the arms, $p\left(g_i, g_j\right)$ denotes the probability that agent *i* becomes the winner to claim the entire resource. We employ the following function form for the technology of conflict.[5]

$$p\left(g_i, g_j\right) = \frac{f(g_i)}{f(g_i) + f(g_i)} \tag{1}$$

$f\left(x\right)$ is a non-negative and increasing; i.e., $f'/\left(x\right) \geq 0$. We also assume $f'\left(x\right)^2 > f''\left(x\right) f\left(x\right)$ to satisfy second-order conditions.[6] The symmetry in this conflict technology ensures fair competition. The result of the contest determines the interim winner and loser in the first period.[7]

Second, the loser has an opportunity to appropriate the prize from the winner. This consecutive contest is a battle between offense and defense. We continue to assume here that the battle is the winner-take-all contest such that the final winning agent can have perfect security about her prize. In this sense, payoffs from consuming the prize are realized at the end of the second period.[8] At this point, we

[5]This class of conflict technology has been employed in several studies, as is well summarized in Dixit (1987) and in Garfinkel and Skaperdas (2006). They also explain some of the main characteristics of this technology and compare it to other functional forms.

[6]This model satisfies a sufficient condition for the existence and uniqueness of equilibrium; the proof can be found in Skaperdas (1992).

[7]Indeed, our model is not directly applicable to R&D races, because innovation always takes place in the model of contest. However, we believe that our basic ideas and intuitions are applicable in various types of dynamic competition.

[8]Another way to understand this model is by considering that symmetric agents compete for the prize R_1 in the first period, and depending on the outcome of the first period, the favorable winner and unfavorable loser compete for R_2 in the second period. Our basic model is merely normalizing $R_1 = 0$. In fact, including interim payoffs does not change our basic results. In this sense, our model and results are relevant to explain other types of dynamic competition, even if they are not involved in contests with insecure prizes.

make the distinction between offense and defense such that defending the prize may be more effective than predating it given the same level of arms, or vice versa. We adjust the conflict technology in the following manner,

(2) $$q(w,\, l) = \frac{f(w)}{f(w) + \theta f(l)} \quad \text{and} \quad 1 - q(w,\, l) = \frac{\theta f(l)}{f(w) + \theta f(l)}$$

where $q(w,\, l)$ is the probability that the interim winner can keep the prize when w and l are respectively the winner's defensive weapons and the loser's offensive weapons. Likewise, $1 - q(w,\, l)$ is the probability that the loser can appropriate in the second battle. Note that θ is a parameter that indicates the effectiveness of the offensive weapons against the defensive weapons. If θ is smaller (greater) than 1, offense is less (more) effective than defense. Therefore, θ is a measure of the security of its claim to property once one agent owns the property after the contest of the first period. θ = 0 indicates a perfect property right. As θ increases, the property right becomes less strong. We will often refer to the inverse of θ as the degree of defensibility or security in the paper. Another interpretation of the degree of security is the relative advantage that the interim winner has over her rival in the successive battle. For simplicity, we assume that there is no discount and that the war is not destructive.

III. The Equilibrium Analysis

As usual, our analysis starts from the second period following backward induction. Once again, both battles are assumed to be winner-take-all contests. Regardless of who wins in the first period, the winner in the second period has fully secure property rights to the prize. Thus, the interim winner and loser payoff functions are given by

$$V_W = q(w,\, l)R - w \quad \textit{and} \quad V_L = \left[1 - q(w,\, l)\right] R - l.$$

Each agent chooses the number of arms given its rival's choice. We obtain the following first-order conditions: $\frac{\partial V_W}{\partial_W} = \frac{\partial_q (w,l)}{\partial_W} R - 1 = 0$ and $\frac{\partial V_l}{\partial_l} = -\frac{\partial_q (w,l)}{\partial_l} R - 1 = 0.$ Using (2), the first-order conditions are summarized as follows:

$$\frac{[f(w) + \theta f(l)]^2}{\theta f'(w) f(l)} = \frac{[f(w) + \theta f(l)]^2}{\theta f'(l) f(w)} = R.$$

According to these conditions, the winner and loser choose a symmetrical number of weapons for equilibrium regardless of θ. The symmetrical values of $w^*(\theta, R)$ and $l^*(\theta, R)$ must satisfy

(3) $$\frac{f(w^*)}{f'(w^*)}=\frac{f(l^*)}{f'(l^*)}=\frac{\theta R}{(1+\theta)^2}$$

The given assumption of $f(x)$ ensures that $\frac{f(x)}{f'(x)}$ is non-decreasing. It can be easily shown that $w^*(\theta, R)$ and $l^*(\theta, R)$ are decreasing when $\theta > 1$ and increasing when $\theta < 1$. In other words, they invest in arms most when they are involved in fair competition, i.e., $\theta = 1$. The equilibrium probabilities and payoffs are as follows:

$$q(w^*,l^*)=\frac{1}{(1+\theta)} \quad \text{and} \quad 1-q(w^*,l^*)=\frac{1}{(1+\theta)} \;;$$

$$V_W^*=\frac{R}{1+\theta}-w^*(\theta,R) \quad \text{and} \quad V_L^*=\frac{\theta R}{1+\theta}-l^*(\theta,R).$$

First, it is important to note that $V_W^*=\left(\frac{1-\theta}{1+\theta}\right)R+V_L^*$. If $\theta < 1$, the winner's payoff is greater than the loser's payoff. In addition, it is expected that agents prefer to be the winner in the first battle. If $\theta > 1$, in contrast, the loser's payoff is greater. Agents are expected to prefer to be the loser in the first battle. That is to say, in the first battle, agents are willing to lose and therefore do not fight. In such a case, they wait and fight only once in the second period. Thus, hereafter, it is assumed that $\theta < 1$ for the rest of the discussion. It would be rather sensible to assume that the winner favors protecting his prize against the loser's attempt to steal it.

Given the second period outcome above, at this point we solve the following first-period problem. Each agent maximizes

$$V_i = p(g_i,g_j)V_W^* + [1-p(g_i,g_j)]V_L^* - g_i,$$

where each agent obtains the winner's payoff with probability $p\left(g_i, g_j\right)$ and the loser's payoff with probability $1-p\left(g_i, g_j\right)$. The first-order condition is $\frac{\partial V_i}{\partial g_i}=\frac{\partial p(g_i,g_j)}{\partial g_i}\left(V_W^*-V_L^*\right)-1=0$. The incentive for investments in arms depends crucially on the difference between the winner's payoff and the loser's payoff. The symmetrical equilibrium investments in arms, $g^* = g_i^* = g_j^*$ satisfy

$$\text{(4)} \qquad \frac{f(g^*)}{f'(g^*)} = \begin{cases} \dfrac{V_W^* - V_L^*}{4} = \left(\dfrac{1-\theta}{1+\theta}\right)\dfrac{R}{4}, & \text{if } \theta \le 1, \\ 0, & \text{if } \theta \le 1. \end{cases}$$

One can immediately note that $g^*(\theta, R)$ is decreasing in θ, contrary to the effect of θ on $w^*(\theta, R)$ or $l^*(\theta, R)$. As the prize is more defensible in the future, agents fight more aggressively in the first battle. From (3) and (4) together, we find a well-known implication in the literature which holds that the levels of intensity of competition during the first and second battles are negatively correlated.[9]

Lemma 1 $\dfrac{\partial g^*(\theta,R)}{\partial \theta} < 0$ *and* $\dfrac{\partial w^*(\theta,R)}{\partial \theta} = \dfrac{\partial l^*(\theta,R)}{\partial \theta} > 0$. *As the property becomes less defensible, the first battle becomes less aggressive and the second battle becomes more aggressive.*

Let us analyze how the degree of defensibility influences the agents' overall equilibrium investments in arms, which is $g^*(\theta,R) + \dfrac{w^*(\theta,R) + l^*(\theta,R)}{2}$,

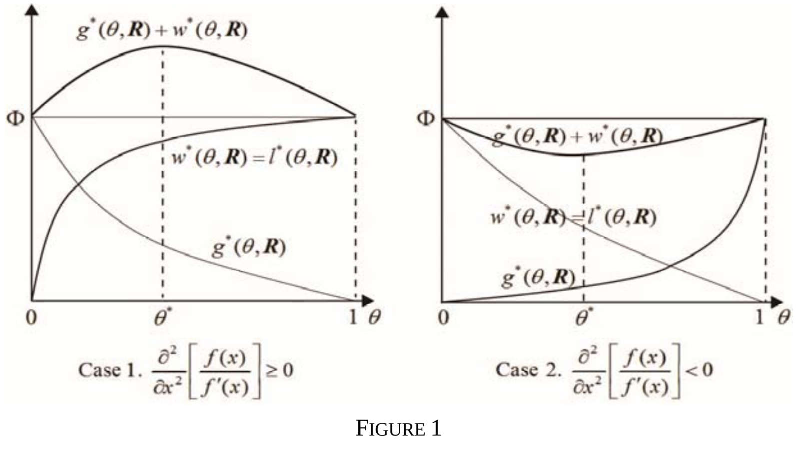

FIGURE 1

[9]This intuitive outcome is comparable to the literature on switching costs and customer poaching competition. When there are switching costs, a group of customers served by a firm in the first period is almost perfectly defensible as its source of profits in the second period. Firms in industries with switching costs compete very aggressively in the initial period, after which then they can attain a collusive outcome. See Klemperer (1987a). However, the literature on customer poaching explains the opposite situation. Roughly speaking, when competing firms expect aggressive customer poaching competition in the second period, they compete less aggressively in the first period. See Fudenberg and Tirole (2000) and Chen (1997).

i.e., $g^*(\theta,R)+w^*(\theta,R)$. We show below that this function is concave and has a maximum of $\theta\in[0,1]$.

Lemma 2 *If* $\dfrac{f(x)}{f'(x)}$ *is convex or linear, there exists a unique maximum* $\theta^*\in[0,1]$. *If* $\dfrac{f(x)}{f'(x)}$ *concave, there exists a unique minimum* $\theta^{**}\in[0,1]$.

The total investments in arms show a non-monotonic relationship in θ, and depend on how the marginal competition changes. If the marginal competition is nonincreasing with regard to the size of the prize, i.e., $\dfrac{\partial^2}{\partial x^2}\left[\dfrac{f(x)}{f'(x)}\right]\geq 0$, the overall competition is increasing as $\theta<\theta^*$ and decreasing as $\theta>\theta^*$.[10] The intuition for this result is as follows. For a relatively large degree of defensibility, the effect of a unit increase in investment in the first period is minimal on one's winning probability or payoff, because competition in the first battle is nearly saturated. The marginal return of investment is greater in the second battle. This is why competition in the second period is marginally more aggressive than in the first. A similar explanation is possible for a relatively small degree of defensibility. In contrast, if the marginal competition is increasing regarding the size of the prize, i.e., $\dfrac{\partial^2}{\partial x^2}\left[\dfrac{f(x)}{f'(x)}\right]<0$, this result dramatically changes in that the overall competition is now decreasing as $\theta<\theta^{**}$ and increasing as $\theta>\theta^{**}$.

In equilibrium, the two agents have an equal probability to receive the prize R, and they spend $g^*(\theta,R)+\dfrac{w^*(\theta,R)+l^*(\theta,R)}{2}$ to buy arms. The symmetric equilibrium payoff of each agent is as follows:

$$V^*(\theta,R)=\begin{cases}\dfrac{V_W^*+V_L^*}{2}-g^*(\theta,R)=\dfrac{R}{2}-\left[g^*(\theta,R)+\dfrac{w^*(\theta,R)+l^*(\theta,R)}{2}\right], & \text{if } \theta\leq 1,\\[2ex] \dfrac{R}{4}, & \text{if } \theta\leq 1.\end{cases}$$

[10] $\dfrac{f(x)}{f'(x)}$ is convex or linear in various types of functions. such as $f(x)=\ln x$ and $f(x)=x^a$ for $0<\alpha\leq 1$.

Example 1 *For a simple example, we assume* $f(x) = x$ *for the technology of conflict. We can easily characterize the equilibrium as follows:* $w^* = l = \frac{\theta}{(1+\theta)^2}R$, $g^* = g_i^* = g_j^* = \left(\frac{1-\theta}{1+\theta}\right)\frac{R}{4}$, *and* $V^* = V_i^* = V_j^* = \frac{R}{4}\frac{3\theta^2+1}{(1+\theta)^2}$. *In this example,* $\theta^* = 1/3$. *Thus, at equilibrium, Payoffs are decreasing if* $\theta < 1/3$ *and increasing if* $\theta > 1/3$.

IV. Policy Implications and Applications

One of the main contributions of this paper is that it shows how the theoretic model and analysis of sequential conflicts presented in the previous section can be applied to a number of social and economic environments, from more direct types such as international arms races to quite subtle types such as R&D races among firms, the designs of procurement auctions, and even educational systems and structures, thereby providing valuable implications pertaining to many types of diplomatic, economic, and educational policies involved therein.

Note that the results summarized above tell us that the advantage created in sequential conflicts characterizes dynamic competition between players and influences the players' payoffs, where the prize the interim winner has obtained is not secure immediately but only has some advantages in later stages of the competition.

The finding that the overall competition or investment levels made by players are crucially affected by the degree of defensibility as well as the direction of changes in the marginal competition regarding the size of the interim prize has important implications on the optimal level of the interim winner's advantage. Whenever such a direction is positive, a contest planner would have to eliminate subsequent competitions which possibly arise thereafter by fortifying the security of the rewards given to an interim winner, if she wants to maximize the overall effort levels of the competing parties. If the direction is the other way around, the planner's strategy should be reversed as well.

The paper introduces a simple but robust result from a contest model, which has only been applied to very restricted areas such as military-relatedness issues. Nonetheless, the results indeed shed light on more extensive economic and social problems, including those shown hereafter. Firstly, there are many economic environments in which these results can have considerable implications. Provided that firms' R&D investment and innovation incentives are regarded as the most important factors for bringing economic development in a number of major technology-intensive industries, from information technology to the biotechnology industries, the optimal length and strength of patent protection to enhance innovation incentives among firms have always been a substantial issue. Similarly, with reference to repeated procurements, the optimal advantages given to incumbent firms and the level of information disclosure/sharing are also critical issues.

Applying the implications of the results is as straightforward as it is useful. For instance, when considering an optimal educational policy as to the college admission process or any similar selection process of which the ultimate goal is to maximize students' effort levels during their pre-college years, the direction of changes in the marginal competition among students may be one of the factors at which the policymakers should closely examine.[11] This paper unfortunately does not show how such information can be acquired, as it is rather an empirical problem to do so. Nevertheless, it does indicate which aspect of students' behaviors should be taken into account and how they can be interpreted and used successfully to achieve policy goals.

Lastly, we provide various types of dynamic competition in which competition in each period is related according to the interim winner's advantage, and discuss the applicability of the results shown above. Unless specified, our discussions are based on the case where $\frac{f(x)}{f'(x)}$ is convex.

- *Insecure Prizes.* Nations and tribes in warfare buy more arms when winner's prize is not perfectly secure. When a sequence of innovations is undertaken by both firms, the firms invest more when the patent for the first innovation is not perfectly strong.
- *Repeated demands for bribes.* When a corrupt government official sells a license which is necessary to open a shop, he commonly demands more than once, threatening with the possibility of the replacement of the license owner.[12] Bribers can anticipate his repeated demands. In such a case, the possibility of a threat in the future is reducing θ from 0 in our model. This aspect can then increase the official's revenue. This result is in sharp contrast to that of Choi and Thum (2003), who studied the dynamics of corruption. In their model, both the dynamic consistency problem and intertemporal price discrimination undermine a corrupt official's revenues. The crucial difference in our model is incorporating competition between bribers.
- *Internal competition at companies.* Meyer (1992) studied biased contests for an organization's promotion decisions. The officer provides employees having shown high performance with more productive tasks, better work environments, or more opportunities for education and training.[13] This result is consistent with choosing the optimal θ^* in our model. However, while that model can show that a positive bias for the first winner minimizes the

[11]Once this factor is specified and clarified, it would greatly help policymakers to reach proper decisions about, for example, whether to allow a college to discriminatorily evaluate applicants according to the performance and reputation of the high school from which they are graduating, as this policy and regulation is closely related to the "interim winner's advantage."

[12]Choi and Thum (2004) introduce several examples or repeated extortion in bribes, organized crime, and expropriations of multinational corporations.

[13]"For example, at 3M, divisions and even groups purposefully compete with one another. At Bloomingdale's, the merchandising vice president, buyers, and fashion coordinators engage in an unending tussle for scarce floor space. The company reorganizes regularly as both winners and losers emerge." — Peters and Waterman (1988)

principal costs, our model represents the uniqueness and magnitude of optimal bias.

- *Learning by doing during repeated procurement.* This issue was studied by Lewis and Yildirim (2002), especially with the example of the repeated purchases of weapon systems, aircraft, and missiles by the Department of Defense. Our model suggests that the principal can prefer handicapping the first winner, who is now more efficient at the reprocurement stage, by information sharing.
- *Property rights and R&D incentives.* Reinforcing property rights is not necessarily desirable for the winner. When $\theta > \theta^*$, her equilibrium payoff decreases as the degree of defensibility rises, i.e., as θ falls. If this is the case, interestingly, contending agents prefer to remain in an insecure situation. The result is reminiscent of Gallini (1992) and Choi (1998) in the imitation literature. They have shown that a longer patent lifetime or a stronger patent does not necessarily increase an innovator's payoff. They came to this result after correspondingly incorporating costly imitation and strategic information transmission through patent litigation. We come to the same conclusion when allowing for repeated innovation by the same firm. Another slight difference is that they investigated *ex-post* innovator's incentives while this paper studies agents' *ex ante* incentives prior to competition.
- *Dynamic competition with switching costs.* The literature on switching costs is comparable to the second case in which marginal competition increases. Klemperer (1987a) basically showed the equivalence of the two extreme cases of $\theta = 0$ and $\theta = 1$. When switching costs exist, the second period competition nearly vanishes in that competing firms are able to achieve a collusive outcome. However, price wars in the first period compete away all of the expected rents in the second period. However, Klemperer (1987b) demonstrated how overall competition could be weakened when there is a group of new consumers in the second period. The portion of new consumers corresponds to θ in our model. They are not defensible for the successive competition as a source of the firms' profits, whereas old consumers whose preferences are constant are locked in. One can find that price competition in his model marginally increases, which is the driving force for the result.[14]

[14]Indeed, the model by Klemperer (1987b) is more elaborate than our simple description above. It has another group of old consumers whose preferences in the second period are independent of their first period tastes. More importantly, consumers' expectations about future prices are crucial to the results. In fact, his focus was on showing that competition in both periods could be relaxed when consumers have rational expectations. Nonetheless, it is true that overall competition is mitigated when consumers are myopic.

V. Extensions and Discussions

In this section we extend the model several ways to realize further implications about sequential conflicts. In each extension, to isolate the implication of each case, we retain the others.

A. *Payoff Advantage*

Thus far, we assume a relative advantage for the interim winner by way of a more effective defense against offense in conflict technology. Here, we study another way by which the interim winner is favored to gain the final payoff. The second competition is now assumed to be fair in that $q(w,\, l) = \dfrac{f(w)}{f(w)+f(l)}$. However, the final winner receives $(1-\theta)R$ securely, while only θR is contestable. The repeated winner is able to obtain an additional gain by $(1-\theta)R$ when she finally wins in the successive contest. Alternatively, this situation can be interpreted as meaning that the loser may be able to appropriate only a portion of the prize. Thus, the inverse of θ continues to represent defensibility. The interim winner and loser's payoff functions are given by

$$V_W = \frac{f(w)}{f(w)+f(l)}\theta R + (1-\theta)R - w \text{ and } \mathrm{V}_L = \left(\frac{l}{w+l}\right)\theta R - l.$$

From the first-order condition and $V_W - V_L = (1-\theta)R,$ it is straightforward to obtain $\dfrac{f(g_i^*)}{f'(g_i^*)} = \dfrac{f(g_j^*)}{f'(g_j^*)} = \dfrac{1-\theta}{4}R.$ Again, only the second battle arises when $\theta = 1,$ whereas only the first battle occurs when $\theta = 1.$ At this point, the expected overall competition is characterized by the following proposition.

Proposition 1 (1) *If* $\dfrac{f(x)}{f'(x)}$ *is convex, there exists a unique maximum* $\theta \in [0,1].$ (2) *If* $\dfrac{f(x)}{f'(x)}$ *is linear,* $\dfrac{\partial}{\partial \theta} = \left[g^*(\theta, R) + \dfrac{w^*(\theta,R)+l(\theta,R)}{2} \right] = 0.$ (3) *If* $\dfrac{f(x)}{f'(x)}$ *is concave, there exists a unique minimum* $\theta^{**} \in [0,1].$

A contest designer can increase contestants' total effort levels by (dis)favoring the first winner during the payoff. Hence, our basic results are robust given this type of advantage. Moreover, this extension provides unique implications about how to allocate the prize in sequential contests. In fact, the setup can be viewed as the contest designer's decision to distribute a single prize by the proportion of

$1-\theta$ and θ in sequential contests. In other words, one can see that $(1-\theta)R$ and θR are merely the payoffs in the first stage and second stage, respectively. In this case, when the marginal competition decreases, a sequential distribution can raise contestants' efforts levels.

This result is worthy of comparison to Clark and Riis (1998), who studied whether to distribute multiple prizes simultaneously or sequentially, and Moldovanu and Sela (2001), who compared the expected sum of efforts by a single prize and by more than two prizes. One important assumption in both papers was that contestants have different abilities. In our model, however, we show that the contest designer can increase the effort level through sequential competition between equally capable contestants.

B. *n-period Model*

One natural question is how the outcome would be if we extend the model to *n*-period contests. When players engage in more battles, do they fight more vigorously? Does a principal want to make sequential contests longer? We answer all of these questions. For analytical simplicity and tractability, we follow the example above for the remainder of the paper.

Suppose that two competing agents fight against each other *n* times to win a prize. The first battle is a fair competition, whereas the successive battles involve protecting and stealing. We solve this game by backward induction. The last *n*-th period game is no more than one in a two-period game. The winner and loser in the $(k-1)$-th period maximizes the following payoff functions, respectively, where $3 \le k \le n.$[15]

$$V_{K-1,W} = q(w_{k-1}, l_{k-1})V^*_{K,W} + (1-q(w_{k-1}, l_{k-1}))V^*_{K,L} - w_{k-1}$$

(5) and

$$V_{K-1,L} = (1-q(w_{k-1}, l_{k-1}))V^*_{K,W} + q(w_{k-1}, l_{k-1})V^*_{K,L} - l_{k-1}$$

Again, the incentive for investments in arms depends on the difference between the winner's payoff and the loser's payoff in the next period. Thus, the number of arms to ensure equilibrium in the $(k-1)$-th period is represented as follows:

$$w^*_{k-1} = l^*_{k-1} = \frac{\theta}{(1+\theta)^2}\left[V^*_{k,w} - V^*_{k,L}\right]$$

Putting this into the payoff functions in (5), we can derive the relationship between the equilibrium payoffs of the $(k-1)$ - th and the *k*-th periods.

[15]We assume for n-period extension that either θ or *n* is not sufficiently large. This guarantees positive payoffs at equilibrium. This is a trivial issue because this assumption can be avoided easily with including interim payoffs.

$$V^*_{K-1,W} - V^*_{K-1,L} = \frac{1-\theta}{1+\theta}\left[V^*_{K,W} - V^*_{K,L}\right] \tag{6}$$

This implies that $\left[V^*_{k\text{-}1,W} - V^*_{k\text{-}1,L}\right] < \left[V^*_{k,W} - V^*_{k,L}\right]$. As a result, the equilibrium level of investments in arms is greater in the successive battle. That is to say, $w^*_{k\text{-}1} - l^*_{k\text{-}1} < w^*_k - l^*_k$. The following proposition summarizes the effects of long periods of battle on the overall competition over n periods and agents' *ex ante* equilibrium payoffs.

Proposition 2 *The overall equilibrium investment in arms is as follows*:

$$g^*_i + \sum_{k=2}^{n} \frac{w^*_k + l^*_k}{2} = \left[\frac{1+\theta}{2\theta} - \left(\frac{1-\theta}{1+\theta}\right)^{n-2}\left(\frac{2+\theta}{4\theta}\right)\right]\left(\frac{1-\theta}{1+\theta}\right)R + \frac{\theta}{(1+\theta)^2}R,\ n \geq 2.$$

As n increases, the overall equilibrium investment in arms rises and agents' payoffs fall.

One implication of this result is that the principal can increase players' effort levels by extending the contest periods. Of course, some costs may be incurred when extending the contest periods. For example, internal competition which is too aggressive hampers cooperative work among workers. Thus, the principal may want to choose the optimal number of periods.

C. *Uncertainty of Defensibility*

The degree of security or defensibility, θ, has been assumed to be certain thus far. Here, we are going to assume it away and study how uncertainty of θ influences agents' arms races and payoffs. Here, θ is a random variable that may follow either $H_1(\theta)$ or $H_2(\theta)$. H_2 entails a riskier second period battle than $H_1(\theta)$ in the sense that $H_2(\theta)$ is the mean preserving spread (MPS) of $H_1(\theta)$.[16]

At this stage, we must solve the maximization problem of expected payoffs for each agent in each period. Risk neutrality allows the expected payoff to be additively separable. Given the functional form, it is straightforward to obtain the following result.

[16]That is, for $\theta \in [0,1]$ and $\int_0^1 \theta h_1(\theta)d\theta = \int_0^1 \theta h_2(\theta)d\theta,\ \int_0^\theta H_1(x)dx \leq \int_0^\theta H_2(x)dx.$

$$E(w^*) = E(l^*) = E\left(\frac{\theta}{(1+\theta)^2}\right)R,$$

(7)
$$E(g^*) = E(g_i^*) = E(g_j^*) = \frac{R}{4}E\left[\frac{1-\theta}{1+\theta}\right],$$

$$E(V^*) = E(V_i^*) = E(V_j^*) = \frac{R}{4}E\left[\frac{3\theta^2+1}{(1+\theta)^2}\right].$$

Proposition 3 *As a situation becomes more risky, the more aggressive the first period battle is, and the less aggressive the second period battle becomes, which is also true for the overall competition.*

The effect of the MPS depends on the concavity or convexity of each of the functions. Given that the values of $w^* = l^*$ are concave in θ, the MPS decreases with regard to its expectation. In contrast, g^* and V^* are convex and their expectation levels increase due to the transformation of the MPS. The intuition behind this result is simple. A more risky battle makes it more important to have a winner's advantage in the second period. Thus, competing agents compete more aggressively in the first battle. In turn, this aggressive competition reduces the intensity of the competition in the second period. In addition, the second period competition dominates the first period competition by $w^* = l^* > g^*$. As a result, agents are better off when they are involved in a more risky battle.

D. *Relaxing the Winner-Take-All Principle*

We have assumed that contests are winner-take-all competitions. This assumption appears to be rather strong and inappropriate in the context of sequential innovation or imitation. However, relaxing this assumption does not change our basic results. In contrast, we find that contending parties compete more aggressively. The intuition here is simple as well. Unless the battle is a winner-take-all competition, they share the prize according to the proportion of $p(g_1, g_2)$ and $1 - p(g_1, g_2)$ after the first battle. In the second battle, they have to protect their own property and attempt to appropriate the rival's property at the same time. Put differently, they compete on two fronts. This makes the first battle more aggressive, making use of relative advantage more on one's front and reducing losses from the relative disadvantage on one's rival's front.

Proposition 4 *The first battle becomes more aggressive if the conflict is not a winner - take - all contest.*

VI. Concluding Remarks

We have developed a simple dynamic model of sequential conflicts. The basic premise of the paper is that contending parties expect ensuing conflicts because properties are not perfectly secure even after engaging in a contest once. We have demonstrated how the degree of insecurity characterizes the dynamic competition in successive contests. We then explored how the contest designer wants to control the relative (dis)advantage of the initial winner in order to maximize all players' overall effort levels, and how the results are related to the several techniques that the contest designer can employ, such as the dividing the prizes, extending the periods of contest, and imposing uncertainty on the security of the prize.

A deficiency of the paper is that it does not analyze social welfare, especially in the context of sequential innovation, because innovation always takes place in our model. For instance, Denicola (2000) studied the socially optimal level of patent protection in an environment of sequential innovation, comparing four different regimes according to whether the second innovation is patentable and infringing. The focus of that paper was to investigate how the patent breadth should be chosen from the perspective of social welfare.[17] In contrast, the present study focuses on characterizing dynamic competition and its implications, being based on more general settings in which various types of sequential conflicts can be analyzed.

Appendix

The Proof of Lemma 2.

Applying the implicit differentiation to (3) and (4), $\frac{\partial}{\partial\theta}\left[g^*(\theta,R)+w^*(\theta,R)\right]\gtreqless 0$ can be rearranged as follows:

$$\frac{F'(w^*)}{F'(g^*)}\lesseqgtr\frac{2(1-\theta)}{(1+\theta)}.$$

Here, $F(x)=\frac{f(x)}{f'(x)}$. Note that the inequality above holds when $\theta = 0$ while the inequality below holds when $\theta = 1$. Because the right-hand side is decreasing in θ, as long as the left-hand side is non-decreasing in θ, θ^* exists and is unique. One can show that the derivative of $\frac{\left[F'(w^*)\right]}{\left[F'(g^*)\right]}$ with respect to θ is non-negative when $F''(x)\geq 0$. The other case can be proven similarly.

[17]Green and Scotchmer (1995) argued that the first patent should be very broad to provide the first innovator with sufficient incentives to invest when different firms undertake a sequence of innovations. However, both this paper and that by Denicola (2000) consider cases in which the same firms compete for repeated innovations.

The Proof of Proposition 1.

The proof here is very similar to that of Lemma 2. $\frac{\partial}{\partial\theta}\left[g^*(\theta,R)+\frac{w^*(\theta,R)+l^*(\theta,R)}{2}\right]\lesseqgtr 0$ can be written as $\frac{F'(g^*)}{F'(w^*)}\gtreqless 1$. Using $g^*(1,R) = w^*(0,R) = 0$ *and* $g^*(0,R) = w^*(1,R) = \Phi$, it is straightforward to show the existence of θ^*. The concave $F(x)$ ensures that $\frac{F'(g^*)}{F'(w^*)}$ is decreasing.

The Proof of Proposition 2.

$$g_i^* + \sum_{k=2}^{n}\frac{w_k^*+l_k^*}{2}$$

$$=\frac{1}{4}\left[V_{2,W}^*-V_{2,L}^*\right]+\frac{\theta}{(1+\theta)^2}\left[V_{3,W}^*-V_{3,L}^*\right]+\cdots+\frac{\theta}{(1+\theta)^2}\left[V_{n,W}^*-V_{n,L}^*\right]+\frac{\theta}{(1+\theta)^2}R$$

Because $\left[V_{k,W}^*-V_{k,L}^*\right]=\left(\frac{1-\theta}{1+\theta}\right)^{n-k}\left[V_{n,W}^*-V_{n,L}^*\right]$ by (6), solving the geometric sequence gives us

$$\sum_{k=2}^{n-1}\frac{w_k^*+l_k^*}{2}=\frac{1-\left(\frac{1-\theta}{1+\theta}\right)^{n-2}}{1-\left(\frac{1-\theta}{1+\theta}\right)}\left[V_{n,w}^*-V_{n,L}^*\right]$$

$$=\frac{1+\theta}{2\theta}\left[1-\left(\frac{1-\theta}{1+\theta}\right)\right]^{n-2}\left[V_{n,w}^*-V_{n,L}^*\right].$$

In addition, $g_i^*=\frac{1}{4}\left[V_{2,W}^*-V_{2,L}^*\right]$ can be rewritten by $\frac{1}{4}\left(\frac{1-\theta}{1+\theta}\right)^{n-2}\left[V_{n,W}^*-V_{n,L}^*\right]$. Thus, we have

$$g_i^*+\sum_{k=2}^{n}\frac{w_k^*+l_k^*}{2}=\left[\frac{1+\theta}{2\theta}-\left(\frac{1-\theta}{1+\theta}\right)^{n-2}\left(\frac{2+\theta}{4\theta}\right)\right]\left[V_{n,w}^*-V_{n,L}^*\right]+\frac{\theta}{(1+\theta)^2}R.$$

Finally, we substitute $\left[V_{n,w}^*-V_{n,L}^*\right]$ by $\left(\frac{1-\theta}{1+\theta}\right)R$.

The Proof of Proposition 3.

Consider any function $v(\theta)$ which is either convex or concave. Via integration by parts twice, we obtain

$$E_{H_1}v(\theta)-E_{H_2}v(\theta)=\int_0^1 v(\theta)d(H_1(\theta)-H_2(\theta))$$

$$=\int_0^1 \left(H_2(\theta)-H_1(\theta)\right)v'(\theta)d\theta$$

$$=-\int_0^1\int_0^\theta \left(H_2(\theta)-H_1(\theta)\right)v'(\theta)d\theta$$

Thus, the sign depends crucially on $v''(\theta)$. $E\left[v\left(\theta\right)\right]$ is greater (smaller) in $H_1\left(x\right)$ if $v(\theta)$ is concave (convex).

The Proof of Proposition 4.

After the first battle, agents 1 and 2 hold $p\left(g_1,\, g_2\right)R$ and $\left(1-p\left(g_1,\, g_2\right)\right)R$, respectively. Hence, each agent engages in offense and defense in the second battle. Following the first-order condition (3), each agent's choice of investment about the property of agent 1 is $w^*=l^*=\frac{\theta}{\left(1+\theta\right)^2}P(g_1,g_2)R$ and that of the property of agent 2 is $w^*=l^*=\frac{\theta}{\left(1+\theta\right)^2}\left(1-P(g_1,g_2)\right)R$. The sum of the two is $\frac{\theta}{\left(1+\theta\right)^2}R$, which is identical to a winner-take-all competition. Each agent's equilibrium payoff in the second battle can be written by follows:

$$V_{2,1}=\frac{1}{(1+\theta)^2}p(g_1,g_2)R+\left(\frac{\theta}{1+\theta}\right)^2\left(1-p\left(g_1,g_2\right)\right)R \text{ and}$$

$$V_{2,2}=\frac{1}{(1+\theta)^2}\left(1-p\left(g_1,g_2\right)\right)R+\left(\frac{\theta}{1+\theta}\right)^2 p(g_1,g_2)R.$$

In the first battle, agent 1 maximizes $p(g_1,g_2)V_{2,1}-g_1$ and agent 2 maximizes $\left(1-p\left(g_1,\, g_2\right)\right)V_{2,2}-g_2$ at the same time. The symmetric outcome is

$g_1^* = g_2^* = g^* = \dfrac{R}{4(1+\theta)^2}$. This is greater than (4) in a winner-take-all competition.

References

Baye, Michael R., Dan Kovenock, and Casper G. de Vries. 1993. "Rigging the Lobbying Process: An Application of the All-Pay Auction." *American Economic Review* 83: 289–294.

Cason, T., W. Masters, and R. Sheremeta. 2010. "Entry into Winner-Take-All and Proportional-Prize Contests: An Experimental Study." *Journal of Public Economics* 94: 604–611.

Choi, Jay Pil. 1998. "Patent Litigation as an Information-Transmission Mechanism." *American Economic Review* 88: 1249–1263.

Choi, Jay Pil and Marcel Thum. 2003. "The Dynamics of Corruption with the Ratchet Effect." *Journal of Public Economics* 87: 427–443.

Choi, Jay Pil and Marcel Thum. 2004. "The Economics of Repeated Extortion." *Rand Journal of Economics* 35: 203–223.

Clark, Derek J. and Christian Riis. 1998. "Competition over More than One Prize." *American Economic Review* 88: 276–289.

Dechenaux, E., D. Kovenock, and R. Sheremeta. 2015. "A Survey of Experimental Research on Contests, All-Pay Auctions and Tournaments." *Experimental Economics* 18 (4): 609–669.

Denicola, Vincenzo. 2000. "Two-stage Patent Races and Patent Policy." *Rand Journal of Economics* 31: 488–501.

Dixit, Avinash. 1987. "Strategic Behavior in Contests." *American Economic Review* 77: 891–898.

Fudenberg, D. and J. Tirole. 2000. "Customer Poaching and Brand Switching." *Rand Journal of Economics* 31: 634–657.

Gallini, Nancy T. 1992. "Patent Policy and Costly Imitation." *Rand Journal of Economics* 23: 52–63.

Garfinkel, Michelle. R. and Stergios Skaperdas. 2006. "Economics of Conflict: An Overview." *Handbook of Defense* Economics, Vol. 2.

Gradstein, Mark, and Kai A. Konrad. 1999. "Orchestrating Rent Seeking Contests." *The Economic Journal* 109: 536–545.

Green, Jerry R. and Scotchmer Suzanne. 1995 "On the Division of Profit in Sequential Innovation." *Rand Journal of Economics* 26: 20–33.

Grossman, Herschel. I. and Minseong Kim. 1995. "Swords or Plowshares? A Theory of the Security of Claims to Property." *Journal of Political Economy* 103: 1275–1288.

Klemperer, Paul. 1987a. "Markets with Consumer Switching Costs." *Quarterly Journal of Economics* 102: 375–394.

Klemperer, Paul. 1987b. "The Competitiveness of Markets with Switching Costs." *Rand Journal of Economics* 18: 138–150.

Laffont, Jean-Jacques and Jean Tirole. 1988. "Repeated Auctions of Incentive Contracts, Investment, and Bidding Parity with an Application to Takeovers." *Rand Journal of Economics* 19: 516–537.

Lewis, Tracy R. and Huseyin Yildirim. 2002. "Managing Dynamic Competition." *American Economic Review* 92: 779–797.

Meyer, Margaret A. 1991. "Learning from Coarse Information: Biased Contests and Career Profiles." *Review of Economic Studies* 58: 15–41.

Meyer, Margaret A. 1992. "Biased Contests and Moral Hazard: Implications for Career Profiles." *Annales d'Economie et de Statistique*: 165–187.

Moldovanu, Benny and Aner Sela. 2001. "The Optimal Allocation of Prizes in Contests." *American Economic Review* 91: 542–558.

Peters, Thomas J. and Robert H. Waterman. 1988. *In Search of Excellence: Lessons from*

America's Best-Run Companies. New York: Warner Books.

Scotchmer, Suzanne. 1996. "Protecting Early Innovators: Should Second-Generation Products be Patentable?" *Rand Journal of Economics* 27: 322–331.

Sheremeta, R. M. 2010. "Experimental Comparison of Multi-Stage and One-Stage Contests." *Games Economic Behavior* 68: 731–747.

Skaperdas, S. 1992. "Cooperation, Conflict, and Power in the Absence of Property Rights." *American Economic Review* 82: 720–739.