

Testing for Nonlinear Threshold Cointegration in the
Monetary Model of Exchange Rates with a Century of Data

Junsoo Lee

(Professor and Rick and Elaine Horsley Faculty Excellence Fellow, Department of
Economics, Finance and Legal Studies, University of Alabama)

Mark C. Strazicich

(Associate Professor, Department of Economics, Appalachian State University)

화폐모형에 의한 환율 결정 이론의 비선형 문턱 공적분 검정:
100년간 자료를 중심으로

이 준 수

(앨라배마대학교 교수)

마크 스트래지시히

(에팔레치안 주립대 부교수)

* 이준수: (e-mail) jlee@cba.ua.edu, (address) Department of Economic, Finance and
Legal Studies, University of Alabama, Tuscaloosa, Alabama 35487-0224, USA
마크 스트래지시히: (e-mail) strazicichmc@appstate.edu, (address) Department of
Economics, Appalachian State University, Boone, North Carolina 28608-2051, USA

- Key Word: Exchange Rates(환율 결정 모형), Threshold Cointegration(비선형공적분),
Monetary Model(화폐모형)
- JEL code: C22, F31
- Received: 2009. 3. 25 • Referee Process Started: 2009. 3. 30
- Referee Reports Completed: 2009. 6. 16

ABSTRACT

The monetary model suggests that nominal exchange rates between two countries will be determined by important macroeconomic variables. The existence of a cointegrating relationship among these fundamental variables is the backbone of the monetary model. In a recent paper, Rapach and Wohar (2002, *Journal of International Economics*) advance the literature by testing for *linear* cointegration in the monetary model using a century of data to increase power. They find evidence of cointegration in five or six of ten countries. We extend their work to the *nonlinear* framework by performing threshold cointegration tests that allow for asymmetric adjustments in two regimes. Asymmetric adjustments in exchange rates can occur, for example, if transactions costs are present or if policy makers react asymmetrically to changing fundamentals. Moreover, whereas Rapach and Wohar (2002) found it necessary to exclude the relative output variable in some cases to maintain the validity of their cointegration tests, we can include this variable as a stationary covariate to increase power. Overall, using their same long-span data, we find more support for cointegration in a nonlinear framework.

환율 결정 모형의 근간이 되는 이론으로 널리 알려져 온 화폐모형은 두 국가 간의 환율이 각국의 통화량과 소득 수준에 의해 결정된다고 설명하고 있다. 그러나 이 이론이 성립하려면 이 모형에 내포된 변수 간에 공적분이 성립해야 하는데, Rapach and Wohar(2002)의 논문은 10개 국가의 자료 중 대 여섯개의 자료에만 (선형) 공적분이 존재한다는 결과를 제시하였다. 본 논문은 그들이 사용한 100년간에 걸친 자료를 사용하되, 환율 결정과정에서 발생할 수 있는 비대칭적

조정과정을 감안하여 비선형 공적분이 성립하는가를 검증하였다. 또한 독립변수가 불안정적이 아닐 경우에는 공적분 관계를 설정하기 곤란하다는 이유로 누락시키는 경우가 많은데 본 논문에서 사용되는 방법론에서는 그러한 문제가 제기되지 않는다. 본 논문에서는 선형 공적분 검정 결과에 비해 더 많은 경우에 있어서 비선형 공적분 관계가 있다는 검정 결과가 산출되었다.

above papers.

To perform our empirical tests, we first consider the ordinary least squares based autoregressive distributed lag (ADL-OLS) threshold cointegration test developed by Li and Lee (2008). We utilize two different threshold effects hypothesized to arise from asymmetric policy responses and/or transactions costs. In particular, we consider threshold models where adjustment to the long-run equilibrium can depend on the *level* or *change* in the deviations from the long-run equilibrium. Moreover, in some countries, the nominal exchange rate and the relative money supply series are each $I(1)$ while the deviation in output series is $I(0)$. While RW (2002) omit the output deviation variable in these cases, we want to include this variable as a stationary covariate in our cointegration tests to increase power. In these cases, we utilize the instrumental variables based autoregressive distributed lag (ADL-IV) threshold cointegration test as suggested in Enders, Im, Lee and Strazicich (2009). The ADL-IV threshold cointegration test is well suited to this task, since the test statistics are unaffected by including stationary covariates.²

Our data set is the same as in RW and consists of over 100 years of annual data on nominal exchange rates (foreign currency per U.S. dollar), national money supplies relative to the U.S. money supply, and real GDPs relative to the U.S. real GDP for fourteen industrialized countries.³ The nominal exchange rate series come from Taylor (2001). The money supply and real GDP data come from Bordo and Jonung (1998) and Bordo, Bergman, and Jonung (1998), respectively. The specific sample periods for each country are reported in our Tables below. Using a long-span data set has the distinct advantage of potentially more observations in each regime and greater power in inference tests. Overall, we find greater support for cointegration in a nonlinear framework as compared to the linear tests. Combining results, we reject the null of no cointegration (in at least one regime) in 8 of the 10 countries examined. These findings provide new support to the growing number of papers by Taylor and Peel (2000) and others who find more support for the monetary model in a nonlinear framework.

The remainder of the paper proceeds as follows. In Section 2, we briefly describe the monetary model and our test methodology. In Section 3, we discuss our empirical findings. We summarize and conclude and Section 4.

² The presence of stationary covariates can pose a problem in existing tests for nonlinear cointegration; see, for example, the papers by Bec, Ben Salem, and Carrasco (2004), Kapetanios, Shin, and Snell (2006), Kapetanios and Shin (2006), and Spagnolo, Psaradakis, and Sola (2005). In our analysis of the monetary model the output gap can be a stationary variable. While we want to include this fundamental variable in our cointegration test to increase power, including stationary covariates in these existing tests will induce a nuisance parameter problem that makes the test statistic dependent on the unknown parameter indicating the signal-noise ratio. As such, these tests are less suitable in our applications. In work not reported here, we additionally examined the monetary model by using the ECM-IV threshold cointegration test as suggested in Enders, Lee, and Strazicich (2007). While the ECM-IV test has similar features as the ADL-IV test, we obtain more rejections of the null using the ADL-IV tests so omit the ECM-IV test results in this paper.

³ We thank David Rapach for generously providing the data.

II. Monetary Model and Testing for Threshold Cointegration

The monetary model can be described by:

$$e_t = \beta_0 + \beta_1(m_t^* - m_t) + \beta_2(y_t^* - y_t) + v_t, \quad (1)$$

where e denotes the nominal exchange rate (foreign currency per unit of domestic currency), m^* denotes the foreign money supply, m denotes the domestic money supply, y^* denotes the foreign country output, y denotes the domestic country output, and t is a time subscript. The United States is the domestic country in each case and all variables are in natural logarithms.⁴ If e_t , $(m_t^* - m_t)$, and $(y_t^* - y_t)$ are each I(1), then the long-run equilibrium condition implies that these variables are cointegrated and $v_t = e_t - \beta_0 - \beta_1(m_t^* - m_t) - \beta_2(y_t^* - y_t)$ will be a stationary process.⁵

While the ADL-OLS threshold cointegration test can have greater power than the ADL-IV test, the ADL-OLS based test has nonstandard distributions that depend on the nuisance parameter when stationary covariates are included. In contrast, the ADL-IV threshold cointegration test is invariant to nuisance parameters in such cases. Therefore, in countries where $y^* - y$, e , and $m_t^* - m_t$ are nonstationary, we will utilize the ADL-OLS threshold cointegration test. Then, in countries where $y^* - y$ is stationary, while e and $m_t^* - m_t$ are nonstationary, we will utilize the ADL-IV test. The ADL-IV based test is well suited in this case, since the same standard normal critical values can be adopted with stationary covariates in the testing equation.

The nonlinear specification of the monetary model in the ADL threshold cointegration test can be described as follows:⁶

⁴ In a strict theoretical framework, the monetary model in (1) predicts that $\beta_1 = \beta_1$ and $\beta_2 < 0$. However, imposing these restrictions is not necessary in our tests for threshold cointegration and may lead to bias test statistics if these restrictions do not strictly hold in practice. As such, we prefer to refrain from imposing any restrictions on these coefficients in our tests. We thank an anonymous referee for bringing this to our attention.

⁵ We note that the model in (1) is one version of the monetary model of exchange rates and there are other versions that have been proposed. In particular, the model in (1) assumes flexible prices and was originally suggested by Frenkel (1976) and Mussa (1976). In this paper, we focus only on the model in (1) since this model has been most often examined in the literature with cointegration tests.

⁶ We allow for threshold effects in the short-run dynamics of the cointegrating model. As an anonymous referee correctly notes, allowing for threshold effects in the cointegrating vector would be an alternative way to capture different regimes in the long-run dynamics. However, allowing for a threshold effect in the long-run cointegrating vector is beyond the scope of the present paper. Instead, we follow the common approach taken in the literature on threshold unit root and cointegration models and allow for threshold effects only in the short-run dynamics; see, for example, the papers by Balke and Fomby (1997), Enders and Siklos (2001), and Hansen and Seo (2002), among others. Presumably, it can be possible to allow for threshold effects in both the long-run and short-run dynamics, but we leave these issues for future research.

$$\begin{aligned} \Delta e_t = & I_t [\rho_1 e_{t-1} + a_1(m_t^* - m_t) + a_2(y_t^* - y_t) + b_1\Delta(m_t^* - m_t) + b_2\Delta(y_t^* - y_t)] \\ & + (1 - I_t) [\rho_2 e_{t-1} + c_1(m_t^* - m_t) + c_2(y_t^* - y_t) + d_1\Delta(m_t^* - m_t) + d_2\Delta(y_t^* - y_t)] \\ & + u_t. \end{aligned} \quad (2)$$

Lags of Δe_t , $\Delta(m_t^* - m_t)$, and $\Delta(y_t^* - y_t)$ can be included as necessary to correct for serial correlations. There are clear advantages to using ADL models; see Li and Lee (2008), and Enders, Im, Lee, and Strazicich (2009) for more details.

Following these methods, we consider two threshold indicators. The first is the so-called threshold autoregressive (TAR) model:

$$I_t = 1 \text{ if } e_{t-1} \geq \tau \text{ and } I_t = 0 \text{ if } e_{t-1} < \tau, \quad (3)$$

where τ is the threshold value. The second threshold indicator is the so-called momentum threshold autoregressive (M-TAR) model:

$$I_t = 1 \text{ if } \Delta e_{t-1} \geq \tau \text{ and } I_t = 0 \text{ if } \Delta e_{t-1} < \tau. \quad (4)$$

We test the following null hypothesis in each case:

$$H_0: \rho_1 = 0 \text{ and } \rho_2 = 0 \quad \text{vs.} \quad H_1: \rho_1 < 0 \text{ and/or } \rho_2 < 0. \quad (5)$$

Thus, under the alternative hypothesis the deviation from the equilibrium will be stationary in at least one regime. We transform the threshold parameter into its percentile and determine this value by minimizing the sum of squared residuals. Specifically, since the threshold parameter cannot be greater or less than the maximum or minimum value of the threshold variable, we first sort the threshold variable e_{t-1} into e_{t-1}^* , which takes the ordered values of e_{t-1} from the minimum to maximum value of e_{t-1} . Then, we consider the following transformation scheme:

$$I_t = I(e_{t-1}^* \geq \tau) = I(\sigma^{-1}T^{-1/2}e_{t-1}^* > \sigma^{-1}T^{-1/2}\tau) = I(\sigma^{-1}T^{-1/2}e_{t-1}^* > c^*), \quad (6)$$

where $c^* = \sigma^{-1}T^{-1/2}\tau$ is the normalized threshold parameter and $\sigma^2 = T^{-1}E(\sum e_{t-1})^2$. Next, we let $e_{t-1}^*(\tau) = c^*$ be the c -th percentile of the empirical distribution of e_{t-1}^* , such that $P[\sigma^{-1}T^{-1/2}e_{t-1}^* \leq c^*] = P[\sigma^{-1}T^{-1/2}e_{t-1}^* \leq e_{t-1}^*(\tau)] = c$. As a result, the threshold parameter τ is transformed into a percentile parameter c defined over the interval 0 and 1, and the asymptotic distribution of the corresponding threshold tests will depend only on the percentile parameter c . We can therefore provide critical values based on the percentile parameter defined on the interval between 0 and 1, rather than on a real value that can potentially vary over $-\infty$ to $+\infty$. We estimate the threshold percentile parameter by a grid search to find the value of e_{t-1} (or Δe_{t-1}) that minimizes the sum of squared residuals from the regression. For the grid search procedure, we use each value of the sorted data of e_{t-1} (or Δe_{t-1}) from the minimum to maximum value, while trimming values at the

<Table 1> ADL-OLS Threshold Cointegration Test Results,
 $I_t = 1$ if $\Delta e_{t-1} \geq \tau$ and $I_t = 0$ if $\Delta e_{t-1} < \tau$

| Country | ρ_1 | ρ_2 | Wald | threshold | percentile | lag |
|-------------------------|----------|----------|-----------|-----------|------------|-----|
| Australia (1880~1995) | 0.019 | 0.320 | 5.552 | -0.055 | 0.147 | 0 |
| Belgium (1880~1989) | -0.143 | -0.037 | 31.292*** | 0.085 | 0.773 | 0 |
| Canada (1880~1995) | -0.315 | -0.079 | 39.505*** | 0.000 | 0.526 | 0 |
| France (1880~1989) | -0.145 | -0.094 | 26.854** | 0.073 | 0.688 | 0 |
| Italy (1880~1995) | -0.239 | -0.124 | 69.056*** | 0.104 | 0.853 | 0 |
| Spain (1901~1995) | -0.185 | -0.193 | 36.351*** | 0.061 | 0.691 | 0 |
| Switzerland (1880~1995) | 0.058 | -0.049 | 15.851 | 0.046 | 0.854 | 0 |
| UK (1880~1995) | -0.128 | -0.086 | 30.427*** | 0.082 | 0.853 | 0 |

Note: The Wald statistic tests the null hypothesis of no cointegration in two regimes ($\rho_1 = \rho_2 = 0$). All models include a constant term without trend. Critical values come from Table 1 in Li and Lee (2008) for the Boswijk version of the ADL-OLS threshold cointegration test with $n = 2$ conditioning variables. The percentile threshold value was determined by minimizing the sum of squared residuals. *, **, and *** denote rejection of the null of no cointegration at the 10%, 5%, and 1% levels of significance, respectively.

lower and upper 10% of the data. Chan (1993) showed that this type of procedure can estimate the threshold consistently under the null and alternative hypotheses. The threshold parameter estimator is super-consistent under the alternative, implying that the estimated value is expected to converge to its true parameter value more quickly under the alternative hypothesis than under the null.

In the ADL-OLS test, we utilize the Boswijk (1994) version of the Wald test to test the null hypothesis as recommended by Li and Lee (2008). The critical values come from Table 1 in Li and Lee (2008). We use critical values corresponding to each of the indicator functions defined in (3) and (4), respectively. In the ADL-IV test we utilize the usual t -statistics to test the significance of ρ_1 and ρ_2 , since these test statistics have standard distributions and are unaffected by including y^*-y as a stationary covariate.

To apply the ADL-IV test we utilize the following instruments:

$$\begin{aligned} w_{1t} &= [I_t(e_{t-1} - e_{t-m}), (1-I_t)(e_{t-1} - e_{t-m})]' \text{ for } [I_t e_{t-1}, (1-I_t) e_{t-1}]', \text{ and} \\ w_{2t} &= [I_t(y_{2,t-1} - y_{2,t-m}), (1-I_t)(y_{2,t-1} - y_{2,t-m})]' \text{ for } [I_t y_{2,t-1}, (1-I_t) y_{2,t-1}]' \end{aligned} \tag{7}$$

where y_{2t} denotes the regressors $[(y_t^* - y_t), (m_t^* - m_t)]'$. We let $w_t = (w_{1t}, w_{2t})'$ for our instrument. The resulting t -test statistics for ρ_1 and ρ_2 will have asymptotic standard normal distributions in each case; see Enders, Lee, and Strazicich (2007) and Enders, Im, Lee, and Strazicich (2009).⁷

⁷ As noted by an anonymous referee, the IV type tests will be biased if the order of integration in the variables is mis-specified, especially if non-stationary variables are incorrectly considered to be stationary since instruments are required on all nonstationary variables. However, using instruments on stationary

I . Introduction

The monetary model suggests that nominal exchange rates between two countries will be determined by important macroeconomic fundamentals. Two early references to the model are Mussa (1976) and Bilson (1978). While the monetary model is intuitively appealing, empirical support for the model is often difficult to find. Perhaps most critical in this regard are the findings in Meese and Rogoff (1983), where the authors obtain better forecasts of nominal exchange rates in a simple random walk as compared to the monetary model. If the monetary model is valid and the fundamental variables are nonstationary, then a cointegrating relationship must exist. Many empirical studies, however, fail to find support for (linear) cointegration in the monetary model (e.g., Meese, 1986, Baillie and Selover, 1987, and Sarantis, 1994). More recently, Rapach and Wohar (2002, RW) advance the literature by performing (linear) cointegration tests of the monetary model using a century of data. By using long-span data to increase power, RW find greater support for the monetary model than in many previous tests and find evidence of cointegration in 5 or 6 of 10 countries.¹

In this paper, we re-examine the long-span data in RW and perform nonlinear threshold cointegration tests. If the underlying model is nonlinear and linear cointegration tests are adopted, then lower power can result. As such, it is possible that greater support for cointegration will be found when adopting nonlinear tests. In this regard, a growing number of recent studies document evidence of nonlinear dynamics in exchange rates (e.g., Taylor and Peel, 2000, Guerra, 2001, Kilian and Taylor, 2003). Nonlinear dynamics in exchange rate might arise, for example, if reaction to fundamentals and adjustment depends on the *magnitude* or *sign* of the deviation from the equilibrium. For instance, Taylor and Peel (2000) find evidence that deviations in exchange rates from the monetary model follow a nonlinear adjustment process. Although Taylor and Peel (2000) note that a tractable way to model nonlinear adjustment is to adopt a threshold model, they adopt an exponential smooth transition autoregressive (ESTAR) model perhaps for convenience of estimation. While these and other recent papers find greater support for the monetary model in nonlinear models, these papers do not provide formal tests for nonlinear cointegration. Analogous to the linear case, if the variables in a nonlinear monetary model are nonstationary and not cointegrated, then spurious estimates can result. It remains to be seen whether nonlinear cointegration holds or not, but this important question was not examined in the

¹ Rapach and Wohar (2002) initially consider fourteen countries, but some of the countries contain a mix of $I(0)$ and $I(1)$ variables that cannot be cointegrated, and in one country, The Netherlands, all of the variables in the model are $I(0)$ so cointegration tests are not performed for four of these countries. However, in this paper, we utilize $I(0)$ regressors in our testing scheme rather than discarding them as we shall see more details shortly. Thus, our procedure permits us to overcome a limitation of Rapach and Wohar (2002) in this regard.

III. Empirical Results

We now examine the results of testing for threshold cointegration. To be consistent in our comparisons to the linear tests in RW, we utilize their same unit root test results and the same long-span data. In the ADL-OLS tests, we determine the optimal number of lags in the testing regression by employing the Schwarz information criteria (SIC). In the ADL-IV tests, we jointly determine the optimal value of m to construct a proper IV (w_t) and the optimal number of lags to correct for serial correlations. We first search for the optimal lag for a given value of m , for $m = 1$ and $maxm$, where $maxm$ is given as $T^{0.5}$. We then determine the optimal value of m as the value that minimizes the residual sum of squares (RSS) from the regression using the optimal number of lags.

1. Asymmetric Momentum Threshold Effects

We first examine the ADL-OLS threshold cointegration test results with asymmetric effects modeled by the *change* in deviations from the equilibrium in the monetary model. This is the momentum threshold model, where the speed of adjustment to the equilibrium will depend on whether the change in the deviation is above or below the threshold level ($I_t = 1$ if $\Delta e_{t-1} \geq \tau$ and $I_t = 0$ if $\Delta e_{t-1} < \tau$). To obtain valid ADL-OLS test results, we will consider only threshold cointegration tests for the eight countries where e , m^*-m , and y^*-y were each identified as I(1) variables in RW. The test results are displayed in Table 1. Looking at the results, we observe that 6 of the 8 countries reject the null of no cointegration in at least one regime (Belgium, Canada, France, Italy, Spain, and the UK) at the 1% level of significance. Moreover, in each country, except Spain, the speed of adjustment to the monetary model equilibrium is fastest when the rate of depreciation is above the threshold level. Given that the threshold level is close to zero in each case, these findings suggest that nominal exchange rates adjust more quickly to the equilibrium predicted by the monetary model when they are depreciating rather than appreciating. For example, in Canada the estimated persistent parameter when the change in the deviation from the equilibrium is above the threshold level (in regime 1) is -0.315, which is clearly stationary. In contrast, the estimated persistent parameter when the change in the deviation from the equilibrium is below the threshold level (in regime 2) is -0.079, implying that nominal exchange rate behave as a random walk. While less extreme, the differences in the estimated persistent parameters are similar in four of the other five countries that reject the null of no cointegration (Belgium, France, Italy, and the UK). One possible explanation for these findings could be that policy makers are more likely to intervene in currency markets when their currency is depreciating than when their currency is

variables should not lead to any serious bias.

appreciating. This is an example of policy response to different economic conditions; see also Lee (2006) for the case of Korea regarding fiscal policy response to economic cycles.

2. Asymmetric Deviation Threshold Effects

We next examine the ADL-OLS threshold cointegration test results with asymmetric threshold effects modeled by the *level* of the deviations from the equilibrium. This is the autoregressive threshold model, where the speed of adjustment to the equilibrium depends on whether the level of the deviation is above or below the threshold level ($I_t = 1$ if $e_{t-1} \geq \tau$ and $I_t = 0$ if $e_{t-1} < \tau$). Again, to obtain valid ADL-OLS test results we will consider only threshold cointegration tests for the eight countries where e , m^*-m , and y^*-y were each identified as I(1) variables in RW. The test results are displayed in Table 2. Looking at the results, we observe that 4 of the 8 countries reject the null of no cointegration in at least one regime (Canada, Italy, Switzerland, and the UK) at the 1% or 5% level of significance. In two of the four countries (Canada and Italy) that reject the null of no cointegration, the difference in the adjustment speeds is similar to that in the momentum models of Table 1. In Canada, the estimated persistent parameter when the deviation from the equilibrium is above the threshold level (in regime 1) is -0.710 while the estimated persistent parameter when the deviation is below the threshold level is -0.310. In Italy, the

**<Table 2> ADL-OLS Threshold Cointegration Test Results,
 $I_t = 1$ if $e_{t-1} \geq \tau$ and $I_t = 0$ if $e_{t-1} < \tau$**

| Country | ρ_1 | ρ_2 | Wald | threshold | percentile | lag |
|-------------------------|----------|----------|-----------|-----------|------------|-----|
| Australia (1880~1995) | 0.019 | 0.320 | 14.136 | 0.404 | 0.853 | 0 |
| Belgium (1880~1989) | -0.125 | -0.071 | 11.246 | 0.165 | 0.809 | 1 |
| Canada (1880~1995) | -0.710 | -0.310 | 30.634*** | -0.038 | 0.241 | 1 |
| France (1880~1989) | -0.248 | -0.194 | 18.065 | -0.050 | 0.422 | 1 |
| Italy (1880~1995) | -0.441 | -0.136 | 53.553*** | 0.342 | 0.853 | 1 |
| Spain (1901~1995) | -0.336 | -0.291 | 21.232 | -0.060 | 0.415 | 1 |
| Switzerland (1880~1995) | -0.317 | -0.317 | 25.908** | -0.054 | 0.272 | 1 |
| UK (1880~1995) | -0.199 | -0.327 | 26.999** | 0.022 | 0.466 | 1 |

Note: The Wald statistic tests the null hypothesis of no cointegration in two regimes ($\rho_1 = \rho_2 = 0$). All models include a constant term without trend. Critical values come from Table 1 in Li and Lee (2008) for the Boswijk version of the ADL-OLS threshold cointegration test with $n = 2$ conditioning variables. The percentile threshold value was determined by minimizing the sum of squared residuals. *, **, and *** denote rejection of the null of no cointegration at the 10%, 5%, and 1% levels of significance, respectively.

estimated persistent parameter when the deviation from the equilibrium is above the threshold level (in regime 1) is -0.441, while the estimated persistent parameter when the deviation is below the threshold level is -0.136. In the other two countries (Switzerland and the UK) that reject the null of no cointegration, the results are less clear. In Switzerland, the adjustment speeds are the same in each regime, while in the UK the speed of adjustment to the equilibrium is fastest when the deviation from the equilibrium is below the threshold level rather than above. Given the lack of a consistent pattern in the estimated threshold values and/or the persistent parameters in these four countries, it is more difficult to provide a general explanation using the levels of the deviations from the equilibrium for the threshold indicator as compared to the results in the momentum models. Overall, we conclude that momentum threshold models provide the clearest and most intuitive evidence of nonlinear adjustments in nominal exchange rates to the equilibrium predicted by the monetary model.

3. Allowing For Stationary Output Deviations

In the two countries where y^*-y is stationary, while e and m^*-m are nonstationary (Finland and Portugal; see RW), RW omit y^*-y to maintain the validity of their (linear) cointegration tests. In contrast, rather than omit y^*-y from our cointegration tests we want to include this fundamental variable as a stationary covariate to increase power. While omitting this stationary variable can be seen as a limitation of the OLS based cointegration tests, this limitation does not occur in the IV based tests. In contrast, the test statistic in the ADL-IV threshold cointegration test that we consider retains an asymptotic standard distribution even when a stationary covariate is included. Our test results are displayed in Table 3.⁸ Looking at the results, we observe that the null of no cointegration is rejected in at least one regime for Finland at the 5% level of significance. Moreover, it is clear that adjustment to the equilibrium is faster when the change in the deviation is above the threshold level (in regime 1) than when the change is below the threshold level (in regime 2). In particular, the estimated persistent parameter is -0.410 when the change in the deviation from the equilibrium is above the threshold level. This indicates that the nominal exchange rate is clearly stationary and supports adjustment to the equilibrium predicted by the monetary model. However, when the change in the deviation is below the threshold level, the estimated persistent parameter is 0.05 and implies that the nominal exchange rate will behave as a random walk. Overall, including the results for Finland, we can reject the null of no cointegration in the momentum model in 7 out of 10 countries at the 1% or 5% level of significance. If we combine these results with those for Switzerland in Table 2, we can reject the null hypothesis of no cointegration in at least one regime in 8 of 10 countries.

⁸ We adopt only the momentum threshold model in this case since this model already gave the greatest number of rejections of the null.

<Table 3> ADL-IV Threshold Cointegration Test Results y^*-y is treated as $I(0)$, $I_t = 1$ if $\Delta e_{t-1} \geq \tau$ and $I_t = 0$ if $\Delta e_{t-1} < \tau$

| Country | | Coeff | t_{ADL-IV} | $t\text{-stat } \rho_1 = \rho_2$ | lag | m |
|----------------------|----------|-------|--------------|----------------------------------|-----|-----|
| Finland (1911~1995) | ρ_1 | -0.41 | -1.90** | -1.80* | 2 | 7 |
| | ρ_2 | 0.05 | 0.38 | | | |
| Portugal (1890-1995) | ρ_1 | -0.03 | -0.54 | -0.80 | 1 | 8 |
| | ρ_2 | 0.04 | 0.59 | | | |

Note: t_{ADL-IV} tests the null hypothesis of no cointegration in the regime against the alternative of cointegration. Asymptotic standard normal critical values are used for the ADL-IV test (-2.326, -1.645, and -1.282 at the 1%, 5%, and 10% levels of significance, respectively). The value of m in the ADL-IV test was chosen from the model with the minimum sum of squared residuals. The percentile threshold value was determined by minimizing the sum of squared residuals. All models include a constant without trend. $t\text{-stat}$ tests the null that $\rho_1 = \rho_2$. *, **, and *** denote rejection of the null of no cointegration at the 10%, 5%, and 1% levels of significance, respectively.

IV. Conclusion

In this paper, we adopt nonlinear threshold cointegration tests to test for cointegration in the monetary model of exchange rates. While previous researchers have estimated nonlinear versions of the monetary model, they were unable to test for cointegration in a nonlinear framework due to nuisance parameter problems in the existing tests. In this paper, we strive to make a contribution towards filling this gap in the literature. To compare results, we utilize the same long-span data that was previously adopted by Rapach and Wohar (2002) to test for linear cointegration in the monetary model. Given that adopting linear tests can lead to lower power if the underlying model is nonlinear, we test for nonlinear cointegration to see if greater support for the monetary model will occur. We first adopt the ADL-OLS threshold cointegration test developed by Li and Lee (2008) and consider two different threshold models. Following this, we utilize the ADL-IV threshold cointegration test developed by Enders, Im, Lee, and Strazicich (2009). The ADL-IV threshold cointegration test has the distinct advantage that we can include relative output as a stationary covariate to increase power, while the test statistic maintains a standard distribution. Overall, we find greater support for cointegration in the nonlinear framework as compared to the linear cointegration tests in Rapach and Wohar (2002). Moreover, our findings suggest that adjustment to the long-run equilibrium predicted by the monetary model is faster when nominal exchange rates are depreciating as compared to when than appreciating. Finally, our findings complement the growing number of papers that find greater support for the monetary model in a nonlinear framework and perhaps help to explain why Meese (1986), Baillie and Selover (1987), and Sarantis (1994), among others, fail to find support for cointegration in a linear framework.

References

- Baillie, R. T., and D.D. Selover, "Cointegration and Models of Exchange Rate Determination," *International Journal of Forecasting*, 8(1), 1987, pp.43~51.
- Bec, F., M. Ben Salem, and M. Carrasco, "Tests for Unit-Root versus Threshold Specification with an Application to the Purchasing Power Parity Relationship," *Journal of Business and Economic Statistics*, 22(4), 2004, pp.382~395.
- Bilson, J., "The Monetary Approach to the Exchange Rate: Some Empirical Evidence," *IMF Staff Papers* 25, 1978, pp.48~75.
- Bordo, M. D. and L. Jonung, "A Return to the Convertibility Principle? Monetary and Fiscal Regimes in Historical Perspective," in A. Leijonhuvud (ed.), *Monetary Theory as a Basis for Monetary Policy*, MacMillan, London, 1998.
- Bordo, M. D., M. Bergman, and L. Jonung, "Historical Evidence on Business Cycles: the International Perspective," in J. C. Fuhrer and S. Schuh (eds.), *Beyond Shocks: What Causes Business Cycles*, Conference Series, Vol. 42, Federal Reserve Bank of Boston, 1998, pp.65~113.
- Boswijk, H. P., "Testing for an Unstable Root in Conditional and Structural Error Correction Models," *Journal of Econometrics* 63, 1994, pp.37~60.
- Chan K. S., "Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model," *Annals of Statistics* 21, 1993, pp.520~533.
- Enders, W., J. Lee, and M. C. Strazicich, "IV ECM Threshold Cointegration Tests and Nonlinear Monetary Policy in Korea," *KDI Journal of Economic Policy (formerly Korea Development Review)* 29(2), 2007, pp.135~157.
- Enders, E., K. Im, J. Lee, and M. C. Strazicich, "New Threshold Cointegration Tests and the Taylor Rule," Working Paper, University of Alabama, Department of Economics, Finance and Legal Studies, 2009.
- Enders, W. and P. L. Siklos, "Cointegration and Threshold Adjustment," *Journal of Business & Economic Statistics* 19, 2001, pp.166~176.
- Engle, R. F. and C. W. J. Granger, "Cointegration and Error Correction Representation: Estimation and Testing," *Econometrica* 94, 1987, pp.1096~1109.
- Frenkel, Jacob A., "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence," *Scandinavian Journal of Economics* 78, 1976, pp.200~224.
- Hansen, B. and B. Seo, "Testing for Two-regime Threshold Cointegration in Vector Error Correction Models," *Journal of Econometrics* 110, 2002, pp.293~318.
- Guerra, Roger, "Fundamentals and Exchange Rates: How About Nonlinear Adjustment?" Working Paper, University of Geneva, Department of Economics, 2001.
- Kapetanios, G, Y. Shin and A. Snell, "Testing For Cointegration In Nonlinear Smooth Transition Error Correction Models," *Econometric Theory* 22(2), 2006, pp.279~303.
- Kilian, L. and M. P. Taylor, "Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?" *Journal of International Economics* 60, 2003, pp.85~107.
- Kremers, J. J. M., Ericsson, N. R., and J. J. Dolado, "The Power of Cointegration Tests," *Oxford Bulletin of Economics and Statistics* 54, 1992, pp.325~348.
- Lee, S. H., "An Evaluation of Fiscal Policy Response to Economic Cycles," *KDI Journal of*

- Economic Policy (formerly Korea Development Review)* 28(2), 2006, pp.1~44.
- Li, J. and J. Lee, "Single-Equation Tests for Threshold Cointegration," Working Paper, University of Alabama, Department of Economics, Finance and Legal Studies, 2008.
- Meese, R. A., "Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates," *Journal of Political Economy* 94(2), 1986, pp.345~373.
- Meese, R. A., and K. Rogoff, "Empirical Exchange Rate Models of the Seventies," *Journal of International Economics* 14, 1983, pp.3~24.
- Mussa, M., "The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy Under a Regime of Controlled Floating," *Scandinavian Journal of Economics* 78, 1976, pp.229~248.
- Rapach, David E. and Mark E. Wohar, "Testing the Monetary Model of Exchange Rate Determination: New Evidence from a Century of Data," *Journal of International Economics* 58, 2002, pp.359~385.
- Sarantis, N., "The Monetary Exchange Rate Model in the Long-run: An Empirical Investigation," *Weltwirtschaftliches Archiv* 130, 1994, pp.698~711.
- Spagnolo, F., Z. Psaradakis and M. Sola, "Testing the unbiased forward exchange rate hypothesis using a Markov switching model and instrumental variables," *Journal of Applied Econometrics*, 20(3), 2005, pp.423~437.
- Taylor, Alan M., "A Century of Purchasing-Power Parity," *Review of Economics and Statistics* 84, 2002, pp.139~150.
- Taylor, M. P. and D. A. Peel, "Nonlinear Adjustment, Long-Run Equilibrium and Exchange Rate Fundamentals," *Journal of International Money and Finance* 19, 2000, pp.33~53.
- Zivot, E., "The Power of Single Equation Tests for Cointegration When the Cointegrating Vector is Prespecified," *Econometric Theory* 16, 2000, pp.407~439.