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Addressing the Demographic Decline in South Korea†

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In this paper, we use an overlapping-generations standard-incomplete markets model to quantitatively investigate the long-run implications of Korea's demographic changes and policy reforms. Importantly, our quantitative model endogenizes the retirement decision and matches the elasticity of retirement to wealth. We use the model calibrated to Korea's economy and demography as a quantitative laboratory to investigate two policy scenarios: increasing taxes or decreasing benefits. While decreasing benefits leads to greater long run activity, it comes at the cost of lower average welfare, particularly for retirees.

Key Word: Aging, Social Security Reform, Endogenous Retirement, Fiscal Sustainability
JEL Codes: H55, J11, J26, E62

I. Introduction

South Korea continues to have the lowest fertility rate in the world and has for quite some time. Though it has not always been the lowest in the world, Koreans have consistently been born at rates below replacement level. In fact, since around the turn of the century, Korea's old-age dependency ratio has continued to increase, and is projected to overtake Japan around 2050 (Figure 1). Shortly after doing so, Korea's old-age dependency ratio will likely pass one, meaning there will be fewer people in the working-age population than those who are older than working age. In addition to the challenges for economic growth that come with an aging population, an aging population like Korea's presents a significant threat to the nation's fiscal health.

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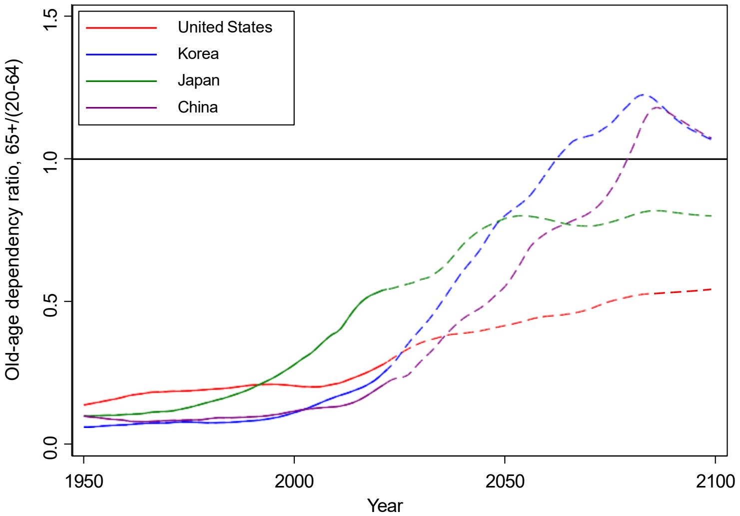


FIGURE 1. OLD-AGE DEPENDENCY RATIO

Note: The old-age dependency ratio is defined as the number of people aged 65+ divided by the number of people aged 20-64. Data are derived from the UN World Population Prospects. Data past 2023 are projections, represented by dashed lines in the chart.

As in many advanced economies, Korea offers a government-sponsored pension plan as part of its social security system. Under this system, workers and firms contribute a portion of their income and payroll expenditures, respectively, to fund the pension. Once a worker reaches retirement age, they receive monthly payments from the pension system that depend on their previous earnings. As the population ages, the number of retirees swells, while the number of working-age individuals declines. Hence, a smaller and smaller number of people are responsible for funding a larger and larger pension fund. Korea's rapidly aging population implies an increasingly large fiscal burden resulting from their retirement pension over the next few decades. Without reforms to the policy system itself, the tax system that funds it, or without generating revenue from other means, deficits caused by the pension program over the next few decades could threaten Korea's overall fiscal health.

In this paper, we use an overlapping-generations standard-incomplete markets model to quantitatively investigate the long run implications of Korea's demographic changes under several reforms to the pension system. We calibrate the model to Korea's economy and demography and use it as a quantitative laboratory to investigate policy scenarios. Because households respond to changes in incentives generated by fiscal reforms, the budget calculus surrounding the public pension depends on the nature of the reform. Our model has two important features that confront this issue. First, we solve the model in general equilibrium. Changes in wages and in aggregate effective labor supply affect the tax revenues available to the government. Second, our model endogenizes the retirement decision. Policies that protect benefits while taxing workers incentivize more retirements—increasing the social cost of providing a public pension. For this reason, we take care to match the elasticity of retirement to wealth.

We present stylized facts in this paper that characterize retirement decisions in

Korea. We show that, all else being equal, a unit increase in log income leads to a two percentage point reduction in the probability of retirement. Similarly, a unit increase in log wealth leads to a one percentage point increase in the probability in retirement. Furthermore, in our quantitative setting, we contrast potential reforms to the Korean retirement system. In particular, we show that raising taxes is preferred to cutting benefits, though both reforms are welfare-reducing. We additionally demonstrate that cutting benefits generates long-run economic growth but is offset by significant retiree welfare losses.

There is a large body of literature that uses quantitative general equilibrium models with overlapping generations to study the implications of potential public pension reforms. Following the seminal work of Auerbach and Kotlikoff (1987), Imrohoroglu *et al*. (1995) allows for uncertain lifetimes, borrowing constraints, and idiosyncratic wage risk over the life cycle. Their main finding is the desirability of a small-sized unfunded social security system in the long run, in part owing to the removal of the dynamic inefficiency induced by it. Conesa and Krueger (1999) extend this work to compute the transition path across an initial steady state with an unfunded social security system and a final steady state in which there is full privatization. A large fraction of individuals are better off remaining in the unfunded system, displaying a significant status quo bias and highlighting the large transitional costs of the privatization of social security.

Huggett and Ventura (1999) focuses on the distributional impact of social security reforms. In particular, they address how consumption, leisure, and welfare will be distributed across households when an unfunded pension system is compared with a two-tiered alternative that allows for a smaller guaranteed unfunded system and a mandatory saving/defined contribution system. Current generations are against the transition to the newer pension system despite the large benefits to future generations and younger cohorts.

Huang *et al*. (1997) is one of the first papers that computes equilibrium transition paths across steady states and calculates two pension reform exercises. In the first computation, an unfunded social security system is suddenly terminated but all generations expecting to receive benefits are compensated by a one- time increase in government debt, which is eventually eliminated in the coming decades by a higher labor income tax rate. This entitlement debt is calculated to be 270% of GDP. In the second exercise, the unfunded pension system is allowed to continue while the government raises additional revenues by a higher labor income tax rate to eventually build a sovereign fund sufficiently large so that returns from this public fund cover annual social security payments.

De Nardi *et al*. (1999) extends Huang *et al*. (1997) and allows warm-glow preferences, an endogenous labor supply, and a link between the labor supply and how retirement benefits are related to past earnings. More importantly, unlike in most earlier models, De Nardi *et al*. (1999) incorporates realistic cohort dynamics by using time-varying cohort sizes and population growth rates projected by the Social Security Administration. This time variation in future fertility and mortality expectations cause a transition from an initial balanced growth path to a final balanced growth path far in the future. Equipped with actual U.S. demographic projections, they quantify the welfare effects of potential reforms along the transition paths.[1]

Attanasio *et al*. (2007) categorize developed and developing economies in a two-region model of the world economy and use population projections by the World Bank to quantify the effect of aging and potential pension reforms on these regions. Their work explores the degree to which pension reform effects differ in closed versus open economy models. Their computations yield quantitatively similar effects of pension reforms on regions but have implications with regard to capital flows across regions, with pension privatization generating a saving glut in the south, pushing real interest rates even lower.

Imrohoroglu and Kitao (2012) allow for both an extensive and intensive margin for labor choice in addition to making benefit claiming an endogenous choice in a quantitative general equilibrium model. They find that cutting benefits (with matching payroll tax relief) or lifting the normal retirement age delivers greater solvency gains for social security and encourages older age work; simply pushing back the earliest claiming age is fiscally and macroeconomically insignificant. Their work suggests that designing reforms that target both benefit generosity and labor supply incentives is important for the long-run sustainability of social security.

To study fiscal sustainability for the rapidly aging Japanese economy, both the standard growth model and overlapping generations have been used. Imrohoroglu and Sudo (2011), Hansen and Imrohoroglu (2016) and Hansen and Imrohoroglu (2023) use versions of the Hayashi and Prescott (2002) model to quantify the effects of financing projected increases in government purchases and transfer payments in Japan with various taxes. Their estimated tax increases are very large, suggesting that alternative and multiple other fiscal tools will be necessary to address the issue of fiscal sustainability. Ihori *et al*. (2011), Okamoto (2013), Braun and Joines (2015) and Kitao (2015), on the other hand, use calibrated general equilibrium models with overlapping generations to study the impact of aging on the Japanese economy. A general theme that emerges from this literature is that benefits will have to be reduced or taxes raised significantly to achieve fiscal sustainability in Japan.[2]

Japan, having aged earlier and more gradually, offers a comparative lens. Its public pension system — comprising the National Pension (NP) and Employees' Pension Insurance (EPI) — has implemented a range of institutional reforms over the past two decades. These include macroeconomic indexing, automatic stabilizers, and actuarial recalibration to align benefits with demographic and economic realities. As a result, Japan has been able to stabilize public pension expenditures at around 10% of GDP despite having one of the highest elderly populations globally. The looming fiscal issue comes from public health expenditures, especially from long-term care.[3]

In contrast, the Korean pension seems to be at a crossroads. Without reforms, spending on the public pension scheme would rise rapidly. By the 2060s, Korea's old age (65 years or older) population is projected to exceed the working age (between 20 and 64) population. Given these projections, Kim (2013) calls for a

[1]Also see Krueger and Kubler (2006), McGrattan and Prescott (2017), Hosseini and Shourideh (2019), and Makarski *et al.* (2024).

[2]For an extensive analysis of pension reform effects under aging in Spain, see Diaz-Gimenez and Diaz-Saavedra (2009).

[3]The fiscal effects of aging in Japan have recently been somewhat less burdensome, especially with regard to pensions, due to the reductions in the replacement rate and increases in the payroll tax rate initiated with the 2004 'macroeconomic slide' reform; see Imrohoroglu *et al.* (2016).

round of reforms — balancing adequacy with sustainability — shortly. In particular, Kim (2013) points to the use of political feasibility and intergenerational equity as guiding principles, and suggests harmonizing occupational schemes, strengthening private pension governance, making potential increases in the contribution rate, setting the retirement age beyond 65, and making adjustments to the accrual formula to calculate pensions.[4]

Recently, Baksa *et al*. (2024) looked at potential pension reforms. Similar to the studies on Japan, they argue that using a single fiscal lever to achieve fiscal sustainability is likely not feasible given the anticipated increase in pensions by four percentage points of GDP between 2020 and 2070 if benefits remain unchanged. Raising the retirement age only would require going from 65 to 71, raising the contribution rate only would necessitate a 14-percentage point increase and reducing the benefits by ten percentage points. A combination of raising the retirement age to 67, raising the contribution rate by 4.6%, and reducing the replacement rate by 3.3% brings sustainability.

The remainder of the paper is structured as follows. Section 2 presents stylized facts pertaining to retirement behavior in Korea using data from the Korean Longitudinal Study of Aging (KLOSA) and the retirement pension system. Section 3 describes our quantitative model, while Section 4 details the calibration strategy. Section 5 presents the quantitative results of our policy experiments, focusing on the macroeconomic and welfare effects of different reform scenarios. Finally, Section 6 concludes and provides directions for future research.

II. Stylized Facts

A. *Transition to Retirement*

Individuals chose to retire at some point during their life cycle based on their levels of income, wealth, their expected income after retirement, in addition to other factors such as health. The benefits received from a government pension system play a crucial role. An essential part of our analysis is to understand how an individual's income, wealth, and expectations about retirement income change their retirement decision.

Using data from the Korean Longitudinal Study of Aging (KLoSA), we approximated the probability of transitioning from a worker to a retiree based on income, wealth, and other salient factors (Korea Employment Information Service, 2023). The KLoSA panel tracks around 10,000 Korean individuals between 2006 to 2020 every two years who entered the survey as a worker and either remained a worker or retired. In the survey, the worker/retiree status is self-reported and retirement is defined as retired with no intention of working "unless circumstances change." We measured income at the person level and wealth at the household level. Summary statistics of these data are available in Table A1.

To predict the probability of transitioning to retirement conditional on real income

[4]Kim and Lee (2021) use Korea's National Transfer Accounts to examine the effects of population aging and argue that the public pension system will face significant fiscal pressure in the future.

and real wealth from the previous period, we fitted the following logistic regression model, which predicts retirement, $R_{it} \in \{0,1\}$ in time, t, for person, i,

$$(1) \qquad logit(R_{it}) = \ln\left(\frac{\Pr(R_{it})}{1-\Pr(R_{it})}\right) = \alpha_{it} + \eta_{it} + \phi_t + \beta_y y_{it} + \beta_a a_{it} + \sum_{j=1}^{J} \psi_j x_{jit},$$

where y_{it} is the log of after-tax income reported for the previous year, a_{it} is the log of household wealth reported for the previous year, and x_{jit} represents a variety of additional covariates, identified by j. These covariates can include quadratic log income and log wealth terms, the interaction between log income and log wealth, forward public transfer income, and the average number of hours worked per week. We included age fixed effects, α_{it}, defined as the current age group of the respondent (intervals of two between 50–70, less than 50, and over 70). Additionally, we added survey year fixed effects, ϕ_t, and self-reported health fixed effects, η_{it}. The survey respondents are able to report their health as excellent, very good, good, fair, or poor.

To interpret the estimated regression coefficients, we derive the average marginal effect (AME) of both income and wealth on the probability of retirement from Equation 1, which can be approximated as follows,

$$(2) \qquad \mathbf{E}\left[\frac{\partial \Pr(R_{it} \mid y_{it}, a_{it}, x_{it}, \alpha_{it}, \eta_{it}, \phi_t)}{\partial a_{it}}\right] \approx \frac{1}{N}\sum_{i=0}^{I}\sum_{t=0}^{T_i}\frac{F(y_{it}, a_{it}+\epsilon, x_{it}, \alpha_{it}, \eta_{it}, \phi_t) - F(y_{it}, a_{it}, x_{it}, \alpha_{it}, \eta_{it}, \phi_t)}{\epsilon},$$

where $\epsilon > 0$ is sufficiently small, I is the total number of respondents, T_i is the total number of periods per respondent, N is the total number of observations, and

$$(3) \quad F(y_{it}, a_{it}, x_{it}, \alpha_{it}, \eta_{it}, \phi_t) = \frac{\exp\left(\alpha_{it} + \eta_{it} + \phi_t + \beta_y y_{it} + \beta_a a_{it} + \sum_{j=1}^{J}\psi_j x_{jit}\right)}{1+\exp\left(\alpha_{it} + \eta_{it} + \phi_t + \beta_y y_{it} + \beta_a a_{it} + \sum_{j=1}^{J}\psi_j x_{jit}\right)}$$

Using the KLoSA data, we are required to exclude a number of observations from this estimation and perform a variety of data manipulations. First, we dropped those who report never having been a worker or part of the labor force. We also dropped those who are self-employed or have weak labor marker attachment. We defined weak labor market attachment as working less than ten hours per week or if total labor earnings are sufficiently low to rule out being a full-time worker. We also excluded those who either report being retired in all of their survey responses or listed a retirement year that is before their first survey response. Furthermore, we winsorized all variables relating to income, wealth, and hours worked at the one percent level to limit bias from outliers or misreporting.

TABLE 1—RETIREMENT TRANSITION MODELS

	Model 1	Model 2	Model 3	Model 4	Model 5
Fwd. retirement indicator					
Log income	-0.2295***	-3.0369***	-3.0812***	-3.1235***	-2.9351***
	(0.0019)	(0.0105)	(0.0106)	(0.0106)	(0.0112)
Log wealth	0.1561***	-0.0885***	-0.1760***	-0.1888***	-0.1882***
	(0.0009)	(0.0032)	(0.0046)	(0.0046)	(0.0046)
Quadratic log income		0.4451***	0.4216***	0.4272***	0.3994***
		(0.0017)	(0.0019)	(0.0019)	(0.0020)
Quadratic log wealth		0.0245***	0.0228***	0.0237***	0.0232***
		(0.0003)	(0.0004)	(0.0004)	(0.0004)
Log income × log wealth			0.0346***	0.0358***	0.0366***
			(0.0014)	(0.0014)	(0.0014)
Fwd. Public transfer income				-0.0213***	-0.0209***
				(0.0012)	(0.0012)
Avg. weekly labor hours					-0.0040***
					(0.0001)
Observations	15,492,663	15,492,663	15,492,663	15,476,301	15,476,301
Age FE	Yes	Yes	Yes	Yes	Yes
Survey Year FE	Yes	Yes	Yes	Yes	Yes
Health FE	Yes	Yes	Yes	Yes	Yes
Log income mean (SD)	3.04 (.76)	3.04 (.76)	3.04 (.76)	3.04 (.76)	3.04 (.76)
Log wealth mean (SD)	4.86 (1.6)	4.86 (1.6)	4.86 (1.6)	4.86 (1.6)	4.86 (1.6)
Log income AME [95% CI]	-.02 [-.02,-.02]	-.02 [-.02,-.02]	-.02 [-.02,-.02]	-.02 [-.02,-.02]	-.02 [-.02,-.02]
Log wealth AME [95% CI]	.01 [.01,.01]	.01 [.01,.01]	.01 [.01,.01]	.01 [.01,.01]	.01 [.01,.01]

Note: Asymptotic standard errors in parentheses. * p< 0.10, ** p< 0.05, *** p< 0.01. Reported coefficients are untransformed. The reported AME statistics represent the average change in retirement probability associated with a one unit increase in log income or log wealth.

Table 1 reports the results of an estimated equation (1) and the derived AMEs following equation (2). We estimated that a one unit increase in log income and log wealth is associated with a two percentage point reduction and a one percentage point increase, respectively, in the probability of retiring in the next period, *ceteris paribus*. These effects are statistically significant, but relatively small. A one-unit log increase in income and wealth is large, yet the probability of retirement only changes marginally. Therefore, Korean workers tend to make retirement decisions mainly due to other factors, a feature that will be present in our quantitative model.

These results are robust to including a variety of additional covariates and nonlinear terms. We additionally fit probit and Tobit versions of the model to validate these results, finding no meaningful differences across model specifications. These alternate estimator specifications are reported in Table A2. The age-group fixed effects are an important part of this specification. Intuitively, two individuals with equal income and wealth would not have the same probability of retiring if one individual is 45 years old and the other is 65 years old.

B. *Korea's Retirement System*

Korea's social security system consists mainly of the National Pension scheme (NPS) and the Basic Pension. The Basic Pension is a transfer provided to low-income individuals 65 years of age or older. The focus of this paper is the National Pension scheme, which covers a larger fraction of the population and involves larger transfers than the Basic Pension.

The National Pension is available for individuals 60 years of age or older (to be raised gradually to age 65 from 2011 to 2033) with at least 20 years of contributions. Reduced amounts are available for those 55 years of age or older and those who have made a minimum of ten years of contributions. The income replacement rate for the average income earner who has contributed for 40 years is gradually declining from 50 percent in 2008 to 40 percent by 2028.

The NPS benefit is a function of the average annual income of each participant (with minimum and maximum caps), y, the average income of all participants over the past three years, Y, and the number of years contributed (requiring a minimum of ten to qualify), n. The annual benefit formula is expressed as follows:

$$b = \alpha \frac{Y + y}{2} \times \frac{0.05n}{2} \tag{4}$$

where α governs the replacement rate and is set to 0.40 so that the income replacement rate is 40 percent for the average income earner who has contributed for 40 years.

The minimum and maximum caps on the calculation of average income are KRW 390,000 and KRW 6,170,000 per month, respectively, or equivalently ten percent and 165 percent of average income, Y.[5] Thus, in the absence of growth, the retirement benefit for any individual 65 years of age or older with more than ten years of contributions can be expressed as

$$\frac{b(e_T)}{Y} = 0.005 e_T, \tag{5}$$

where T is the period of retirement and

$$e_{t+1}^{i} = e_t^{i} + 1 + \max\left\{0.1, \min\left\{1.65, \frac{y_t^{i}}{Y}\right\}\right\} \tag{6}$$

where y_t^i denotes annual earnings of individual i. The total contribution rate is nine percent, with half being paid by the employer for workplace-based insured individuals.

[5]See https://www.nps.or.kr/jsppage/english/scheme/scheme_02.jsp

III. Model

Households have a life cycle as in the perpetual youth models of Yaari (1965) and Blanchard (1985). A household is born as a worker with assets, a, and labor productivity, ε. Over its working life, a household faces idiosyncratic labor productivity risk against which it can self-insure by accumulating non-state-contingent assets. Each period, an old worker chooses whether to retire permanently or to continue working, subject to preference shocks drawn from a Type-I extreme value distribution, an assumption that is common in dynamic discrete choice models. Once the household has retired, its retirement benefit is determined by lifetime earnings, e. The retired household finances its consumption from its retirement benefit and the wealth it accumulated during its working life. Each period, it has a probability, δ_R, of dying. When a retired household dies, its assets and productivity level are transferred to a newborn worker household.

Worker households supply ℓ_Z efficiency units of labor, where ℓ and z denote hours supplied and labor productivity, respectively. We assume that z follows a Markov process with transition matrix $\Gamma(z,z')$. Let $y = w\ell z$ denote a household's pre-tax earnings. The government taxes earnings at a flat rate τ_t, which funds the retirement benefits, $b_t(e)$, for retirees.

Let $j \in \{Y,W,R\}$ denote whether a household is a young worker, a worker, or a retiree. A retiree household with wealth a and lifetime earnings e in period t solves

$$
\begin{aligned}
\text{(7)} \qquad & V_t^R(a,e) = \max_{c,a'} u(c) + \beta\left[(1-\delta_R)V_{t+1}^R(a',e) + \delta_R\Omega(a')\right] \\
& \text{s.t. } c + a' \le (1+r_t)a + b_t(e) \\
& a' \ge 0.
\end{aligned}
$$

where r_t is the return on assets, $b_t(e)$ is the retirement benefit, which depends on lifetime earnings, and Ω reflects 'warm-glow' utility from bequests.

Similarly, the problem of a worker household with wealth, a, productivity, z, and previous earnings, e, in period t can be stated as

$$
\begin{aligned}
\text{(8)} \qquad & V_t^W(a,z,e) = \max_{c,l,a'} u(c,\ell) + \mathbf{E}_{\varepsilon_t}\left\{\max_{j=W,R}\left\{\beta\mathbf{E}_{z'|z}V_{t+1}^j(a',z',e') - \tau_{Wj} + \nu\varepsilon_t^j\right\}\right\} \\
& \text{s.t. } c + a' \le (1-\tau_t)w_t z\ell + (1+r_t)a \\
& a' \ge 0.
\end{aligned}
$$

where g updates lifetime earnings based on previous lifetime and current earnings, similar to Hur (2018) and Kitao (2014), and τ_{ij} is the cost of switching from life stage i to j. $\varepsilon \equiv \{\varepsilon_W, \varepsilon_R\}$ is a pair of idiosyncratic preference shocks, which are realized at the end of period t. They are assumed to be i.i.d. and Type-1 extreme value distributed with a zero mean. In particular,

$$F(\varepsilon) = \exp(-\exp(-\varepsilon - \overline{\gamma})) \tag{9}$$

where $\overline{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x))dx$ is Euler's constant and $f(\varepsilon) = \partial F / \partial \varepsilon$. Note that the $\overline{\gamma}$ adjustment makes the distribution have zero mean. The interpretation of ϵ_t^j should be that they are realized today, at the end of time t, but represent the value of being in life stage j next period, at time $t+1$.

Given our assumption on the preference shocks, we can write

$$\begin{aligned} V_t^W(a,z,e) = \max_{c,l,a'} u(c,\ell) + \nu \log\left\{ \sum_{j=W,R} \exp\left(\beta \mathbf{E}_{z'|z} V_{t+1}^j(a',z',e') - \tau_{ij}\right)^{\frac{1}{\nu}} \right\} \\ \text{s.t. } c + a' \leq (1-\tau_t) w_t z \ell + (1+r_t)a, \\ e' = g(e, w_t z \ell), \\ a' \geq 0. \end{aligned} \tag{10}$$

reflecting the current-period utility and the discounted option value of retirement. The retirement probability can also succinctly be formulated as

$$\rho_t(a',e'|z) = \frac{\exp\left(\beta \mathbf{E}_{z'|z} V_{t+1}^R(a',z',e') - \tau_{WR}\right)^{\frac{1}{\nu}}}{\sum_{k=W,R} \exp\left(\beta \mathbf{E}_{z'|z} V_{t+1}^k(a',z',e') - \tau_{Wk}\right)^{\frac{1}{\nu}}}. \tag{11}$$

Derivations of these expressions are provided in the Appendix. ν governs the elasticity to which retirement depends on economic variables. As $\nu \to \infty$, workers retire for purely random reasons, whereas the opposite is true when $\nu \to 0$.

Finally, a young worker household with wealth, a, productivity, z, and previous earnings, e, in period t can be stated as

$$\begin{aligned} V_t^Y(a,z,e) = \max_{c,l,a'} u(c,\ell) + \beta \mathbf{E}_{z'|z}\left[\delta_W V_{t+1}^W(a',z',e') + (1-\delta_W) V_{t+1}^Y(a',z',e')\right] \\ \text{s.t. } c + a' \leq (1-\tau_t)\Gamma_Y w_t \ell + (1+r_t)a, \\ e' = g(e, w_t \ell), \\ a' \geq 0. \end{aligned} \tag{12}$$

where δ_W is the fixed probability that a young worker transits to a worker household stage and $\Gamma_Y < \Gamma_W = 1$ adjusts for the age profile for average wages.

A representative firm uses effective units of labor and capital to produce according to a constant returns to scale technology $F(N,K)$. Firm optimality conditions imply that factor prices are given by $w_t = F_{Nt}$ and $r_t = F_{Kt} - \delta$, where δ is the depreciation rate of capital. Let $\mu_t^j(a,\varepsilon,e)$ denote the measure of households with life stage $j \in \{Y,W,R\}$, wealth a, productivity z, and previous earnings, e. A competitive equilibrium is household allocations, firm allocations, and prices, such that given prices and taxes, household and firm allocations are optimal, government budget constraints are satisfied, and markets clear:

(13) $$F(K_t,N_t) = \sum_{j\in\{Y,W,R\}} \int \left(c_t^j(a,z,e) + a'_{jt}(a,z,e) - a(1-\delta)\right) d\mu_t,$$

(14) $$N_t = \sum_{j\in\{Y,W\}} \int z\Gamma_j \ell_t(a,z,e) d\mu_t,$$

(15) $$K_t = \sum_{j\in\{Y,W,R\}} \int a'_{jt}(a,z,e) d\mu_t.$$

IV. Quantitative Results

Using our model, we consider a few different policies and reforms that could remedy the issues posed by Korea's aging population. When we feed in a change to death rates such that the model generates a long run change to the old-age-dependency ratio consistent with UN projections, we find that reducing retirement benefits to finance the pension deficit leads to higher long-run GDP and a greater capital stock than does increasing taxes on workers. On the other hand, average consumption is lower when benefits are cut, and overall welfare is lower.

A. *Calibration*

Functional forms.

We assume that worker preferences are separable in consumption and leisure, with their period utility taking the following form:

$$u(c,\ell) = \frac{c^{1-\sigma}-1}{1-\sigma} - \theta_\ell \frac{\ell^{1+\psi}}{1+\psi}.$$

Retirees have the same CRRA utility function over consumption, but in place of a labor disutility term, they derive warm-glow utility from leaving a bequest,

$$\Omega(a') = \theta_1 \frac{\left(a' + \theta_2\right)^{1-\gamma_b}}{1-\gamma_b}.$$

This form follows De Nardi (2004). The parameter θ_1 governs the overall strength of the bequest motive, while the shifter $\theta_2 > 0$ makes bequests a luxury good. We introduce a second source of non-homotheticity by allowing the curvature parameter, γ_b, to differ from the coefficient of relative risk aversion, σ. As discussed in Straub (2019) and Gaillard *et al*. (2023), when $\gamma_b < \sigma$ the marginal utility of consumption declines faster in wealth than does the marginal utility of bequests. This pushes the saving motive out into the right tail of retirees.

Finally, we assume that production takes the form

$$F(K,N) = ZK_t^{\alpha} N_t^{1-\alpha},$$

where the TFP parameter Z scales production to normalize GDP to 1 in the initial steady state.

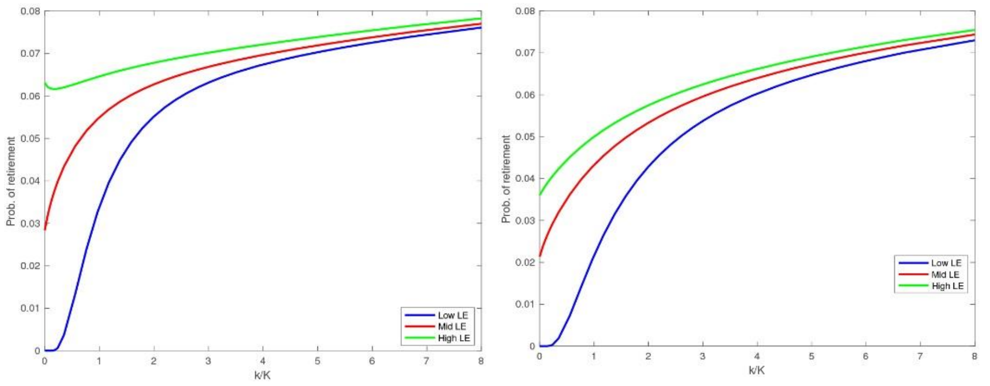
Parameter values.

Following standard values in the literature, we set risk aversion, σ, to 2 and the Frisch elasticity, $1/\psi$ to 0.5. Capital share of output, α, is 0.36, and we assume that capital depreciates at an annual rate of five percent. The fiscal parameters, τ_{SS} and b, are set to 0.09 and 0.005, respectively, based on Korea's national pension scheme. The aging probability of a young worker $\pi_a = 0.05$, and the probability of death for a retiree $\pi_d = 0.05$ such that a household is expected to spend 20 years in each stage. We assume that the labor productivity process follows an AR(1) process in logs, and we follow Chang and Kim (2008) by setting its persistence to 0.80 and the standard deviation of its innovations to 0.354. We also introduce a cost of rebirth equal to 0.5 times average income: when a household dies, the government collects this fee (or the entire estate if it is less than the fee) and uses these revenues to finance the settlement of all estates in the period. In practice, the rebirth cost causes most newborn households to start at low levels of wealth, as in the data.

The remaining seven model parameters are jointly calibrated to match moments from the data. The discount factor β is set to 0.915 to target a wealth-to-GDP ratio of 3.8. The disutility of labor, θ_ℓ, is set so that households spend approximately 39% of their disposable time working (43.8 hours/week). The warm-glow preferences of retirees θ_1, θ_2, and γ_b allow the model to target average retiree wealth of five times average income, the fraction of newborns with near zero wealth, and a wealth Gini of 0.61. Lastly, the variance of retirement shocks, ν, and the psychic cost of retirement, τ_{WR}, are set so that the retirement semi-elasticity with respect to wealth is 1% and households spend 20 years as middle-aged workers on average (so that in the steady state, the average retirement age is 65).

Optimal Retirement Decision.

The retirement decision of a middle-aged working household is a function of its current economic circumstance and an idiosyncratic non-economic shock. While non-economic shocks are orthogonal to the state of the economy, many economic factors influence the worker's decision, including its current and expected future after-tax wages, its present accumulation of private wealth, its total qualified lifetime earnings, and pension generosity. To provide a sense for how these factors alter the decision to retire, Figure 2 plots the initial steady-state retirement probabilities by



(a) Lowest wage (b) Highest wage

FIGURE 2. RETIREMENT PROBABILITY

Note: These figures plot the retirement probability of a middle-aged worker household as a function of wage, lifetime earnings (LE), and wealth (in multiples of mean wealth). The wealth values plotted cover 99.99% of households.

TABLE 2—CALIBRATION

Parameters	Values	Targets / Source
Discount factor, β	0.92	Wealth-to-GDP: 3.8 (OECD)
TFP, Z	1.16	Normalize GDP = 1
Risk aversion, σ	2	Standard value
Disutility from labor, θ_ℓ	20.7	Average hours per worker: 43.8 (OECD)
Frisch elasticity, $1/\psi$	0.5	Standard value
Bequest function,		
Strength of bequest preference, θ_1	9.5	Average wealth of retired households: 5 (KLoSA)
Non-homotheticity parameter, θ_2	2.0	Fraction of young households with non-positive wealth: 30 percent
Curvature on bequests, γ_b	0.75	Wealth Gini: 0.61 (Statistics Korea)
Aging and retirement,		
Probability of old worker, π_a	0.05	Expected young worker years: 20
Retirement disutility, τ_{WR}	60	Expected retirement age: 65
Variance of shocks, ν	20	Retirement semi-elasticity of wealth: 0.01
Death probability, π_d	0.05	Expected retirement years (conditional on 65): 20.7 (2024 Statistics on the Aged)
Old worker wage premium	1.5	Ratio of old to young worker income: 1.5
Capital share, α	0.36	Standard value
Capital depreciation rate, δ	0.05	Standard value
Fiscal parameters		
Social security, τ_{SS}	0.09	National Pension scheme
Replacement rate, b	0.005	Replacement rate for average worker with 40 years of contributions: 40 percent
Wage process,		
Persistence, ρ	0.800	Chang and Kim (2008)
Standard deviation, σ	0.354	Chang and Kim (2008)

wealth for several productivity/lifetime earnings combinations.

The retirement probabilities increase with greater wealth and lifetime earnings and decrease with current wages.[6]

At high levels of wealth, differences in lifetime earnings have a very small impact on retirement probabilities, but they greatly influence the retirement decision for households with below-average wealth (approximately 63 percent of the population).

B. *Empirical Pension System Reform*

As discussed in the Introduction, Korea's dependency ratio is projected to rise substantially over the next century. In order to prevent deficits from expanding, either taxes on workers will need to increase or benefits to retirees will need to decrease. We conduct a set of numerical exercises to examine these reforms. Our main exercise assumes that the death probability, π_d, falls markedly from 5.0 to 2.5 percent, nearly doubling the old-age dependency ratio in the long run. We solve for the steady state arising from each of three fiscal responses. In the first case, the government does not reform the pension system, allowing the deficit to grow unchecked. In the second and third cases, the government adjusts its policy to balance the budget, either by increasing τ or by reducing the replacement rate. For comparison, we also calculate steady states where the government enacts these last two reforms while π_d is unchanged. This allows us to disentangle fiscal reform effects from the effects of an aging population.

The long-run aggregates and factor prices from our exercises are reported in Table 3 along with their values in the calibrated steady state. Under the initial pension, the government runs a deficit of 4.5 percent of GDP. Absent demographic changes, erasing this deficit would require an increase in the tax rate of 5.3 percentage points. Doing this would reduce GDP, capital, and average consumption. Despite a drop in the effective wage, households supply more labor on average. In contrast, reducing benefits promotes economic activity in the long run. Relative to initial levels, GDP rises by five percent, while the capital stock increases by ten percent. Higher wages and smaller pension benefits encourage workers to delay retirement and save more. In equilibrium, the ratio of retirees to workers declines from 0.50 to 0.48. Average consumption falls by 2.4 percent as workers save more and retirees have less pension income.

Imposing demographic change, by lowering π_d, places an enormous burden on the pension system. If the government leaves the status quo in place, the deficit-to-GDP ratio balloons to 11 percent. All major aggregates decline between 20 and 25 percent. Capital shallows, interest rates rise, and wages decline.

Providing for the older population without reducing pension benefits requires an enormous increase in the tax rate from 9.0% to 31.3%. Capital shallows even further, leading to larger movements in factor prices than under no reform. On the other hand, if the government reduces benefits instead, it must cut the replacement rate to 0.17% of lifetime earnings for the average worker with 40 years of contributions. As in the previous case when π_d did not change, reducing benefits to close the deficit

[6]Because productivity shocks are persistent, current wages and expected future wages are correlated.

TABLE 3—AGGREGATES

		τ	b	GDP	K	C	N	r	w	Dep. Ratio	$\frac{Deficit}{GDP}$
Initial SS		9.0%	0.50%	1.00	4.0	0.84	0.36	3.9%	1.77	0.50	4.5%
Aging	$\pi_d =$ 2.5%										
	No Reform	9.0%	0.50%	0.80	3.0	0.76	0.30	4.4%	1.72	0.86	11.0%
	Taxes	31.3%	0.50%	0.78	2.6	0.64	0.31	5.5%	1.61	0.88	0%
	Benefits	9.0%	0.17%	0.93	4.1	0.72	0.32	3.2%	1.85	0.76	0%
No Aging	$\pi_d =$ 5.0%										
	Taxes	16.3%	0.50%	0.99	3.8	0.79	0.37	4.4%	1.73	0.50	0%
	Benefits	9.0%	0.29%	1.05	4.4	0.82	0.37	3.5%	1.82	0.48	0%

Note: This table reports changes from the initial steady in aggregate variables under combinations of demographic changes (π_d = 2.5% or π_d = 5.0%) and fiscal actions.

leads to greater economic activity compared to increasing taxes. GDP falls by only seven percent and capital deepens, leading to higher wages. Once again, by discouraging retirement, the increase in the retiree-to-worker ratio is better contained, reaching only 0.76.

C. *Welfare*

To understand the distributional impact of the three fiscal responses to demographic change described above, we compute household welfare as measured by consumption equivalence. This is defined as the percentage by which a household's initial steady-state consumption would need to change permanently in order to make them indifferent between that steady state and one resulting from reform. Positive (negative) welfare indicates that the reform benefits (harms) that household. Because demographic changes per se may have welfare-reducing effects, we report the welfare difference between each fiscal reform (e.g., raising taxes or lowering benefits) and the 'No Reform' case under which the population ages but the government does not adjust the social security system. Table 4 reports these differences.

TABLE 4—RELATIVE WELFARE DIFFERENCE

(Unit: %)

	Young	Middle	Retired	All
Raise Taxes	-3.0	-1.3	1.6	-0.2
Reduce Benefit	-0.4	-4.2	-31.9	-16.0
Partial Benefit Reduction	-1.4	-0.4	-3.3	-1.9

Note: This table reports the welfare difference relative to the 'No Reform' case with demographic change (i.e., π_d = 2.5%). The three cases correspond to how the government budget constraint is balanced. 'Raise Taxes' achieves this entirely through higher taxes. 'Reduce Benefits' works entirely by lowering the replacement rate. 'Partial Benefit Reduction' lowers the replacement rate by 10% (i.e., from 0.50% to 0.45%) and then raises taxes cover the remainder.

TABLE 5—RELATIVE POPULATION CHANGE

(Unit: %)

	Young	Middle	Retired
Raise Taxes	+0.3	-0.9	+0.6
Reduce Benefit	-1.6	+4.6	-3.0
Partial Benefit Reduction	+0.1	-0.4	+0.2

Note: This table reports the endogenous change in population shares relative to the 'No Reform' case with demographic change (i.e., π_d = 2.5%). The three cases correspond to how the government budget constraint is balanced. 'Raise Taxes' achieves this entirely through higher taxes. 'Reduce Benefits' works entirely by lowering the replacement rate. 'Partial Benefit Reduction' lowers the replacement rate by 10% (i.e., from 0.50% to 0.45%) and then raises taxes cover the remainder.

For the population as a whole, average welfare is considerably greater when the government raises taxes than when benefits are reduced. However, wealth distributions and retirement decisions differ across steady states, meaning that the population weights placed on the welfare changes of various household subpopulations are not invariant across exercises. Table 5 shows how the population shares of each group are affected by the fiscal reform. Raising taxes encourages middle-aged workers to retire sooner, increasing the population of retirees. Reducing benefits has the opposite effect. It strongly discourages retirement, leading to a sizable reduction in the share of retirees.

For this reason, Table 4 also reports the averages within age groups. The age group breakdown makes it immediately apparent that retirees account for the large difference in average welfare across the two reforms. All retirees, regardless of lifetime earnings or financial wealth level, are worse off in a world with reduced benefits compared to one with increased taxes. The reason for this is straightforward: retirees do not pay labor taxes, but they do rely on the replacement rate for their income stream. The differential loss to a retired household decreases with its financial wealth and increases with its lifetime earnings. For very rich retirees, benefit reform has a small impact on their total income, but for most households the reduction is substantial. While raising taxes eases the burden on retirees, it moves it to workers. Young workers have the largest average welfare loss. At the beginning of their life cycle, these households rely on disposable labor earnings for amassing wealth, and higher taxes inhibit this process. Only the very wealthy young households, who derive a small share of their income from supplying labor, support raising taxes.

Middle-aged workers are more mixed. Those with relatively low wages, low wealth, or high lifetime earnings gain more by not cutting benefits. The first two categories need the pension benefits to fund their retirement, while the last group has the largest stake in the public pension and is therefore most exposed to reductions in the replacement rate.

Nevertheless, if steady-state average welfare is a criterion for judging which of the two reforms to enact, taxes should be raised to protect benefits. This raises the question of whether this is a popular fiscal reform. If instead, a vote decided the outcome, would taxes still be raised? To answer this, we check whether a household at state x in the initial distribution would be relatively better off at x in the tax reform steady state than in the benefit reform steady state. Looking over all x in the state space, we calculate the fraction of households who would favor each reform. Consistent with the average steady-state welfare criterion, a popular vote raises taxes and preserves benefits with a 62 percent plurality. Again, all retirees– approximately one-third of the initial population– prefer the tax reform economy, while only six percent of young workers do so. Ultimately, the median voter lies among the middle-aged workers, 82 percent of whom favor increasing taxes.

We also consider a mixed policy response that reduces benefits from their initial level by 10% and then raises taxes to finance the remainder. This approach is designed to spread the costs more evenly over the three age groups. While it does achieve this aim, average welfare is lower than under the ’Raise Taxes’ policy (-1.9% vs. -0.2%). Nevertheless, this mixed approach is more popular, with 67% favoring it over using taxes alone, suggesting that finding a politically feasible solution will likely require sharing the burden across all age groups.

V. Conclusion

This paper quantitatively investigates the macroeconomic and welfare implications of potential social security reforms in Korea using an overlapping-generations model with endogenous retirement. Our findings indicate that while reducing pension benefits promotes long-run economic activity, doing so also generates a significant welfare loss for retirees, who are particularly reliant on pension income. In contrast, raising taxes leads to a smaller reduction in overall welfare. From a political economy perspective, we find that a majority of voters would support raising taxes to preserve benefits, highlighting the political challenges associated with reducing pension benefits despite the potential macroeconomic gains.

We view these exercises as a preliminary investigation into policy reforms in Korea. Our study naturally leads to several areas of future research. First, modeling the life cycle in more detail by replacing life stages with age itself may be important, as this will allow an investigation of the minimum retirement age as an additional policy lever. As the optimal policy likely involves a combination of reforms, such as increasing the retirement age, raising taxes, or making changes to retirement benefits, this larger model will be better suited to mix policy proposals. Second, studying optimal policy reforms along a transition path may be worthwhile given that differences in long-run outcomes may disguise significant countervailing movements along the transition. These transitional costs could be significant, especially in a life-cycle framework where long-run benefits may be unequally spread across generations. Furthermore, considering progressive tax systems and interactions between Korea’s National Pension and Basic Pension could provide additional richness to the model. We leave these potentially interesting extensions for future research.

APPENDIX

A. *Supplementary Tables*

TABLE A1—KLOSA DATA MOMENTS, ALL SURVEYS

	Mean	SD	Min	Max	25th pct.	75th pct.
Workers						
Income	26.6	18.8	1.3	100.0	13.1	35.0
Wealth, HH	217.6	275.9	0.0	1700.0	32.6	286.1
Public transfer income	1.1	3.2	0.0	40.0	0.0	0.0
National pension	0.6	1.9	0.0	18.0	0.0	0.0
Basic retirement benefit	0.1	0.4	0.0	3.4	0.0	0.0
Respondent age	58.2	7.4	45.0	96.0	53.0	62.0
Self-employed income	0.0	0.3	0.0	9.2	0.0	0.0
Hours per week	44.4	13.0	10.0	84.0	40.0	50.0
Wage	23.8	19.9	0.0	92.0	10.9	32.1
Retirees						
Income	2.9	8.7	0.0	100.0	0.0	0.0
Wealth, HH	251.9	321.4	0.0	1700.0	40.8	330.3
Public transfer income	4.2	7.9	0.0	40.0	0.0	4.2
National pension	1.2	2.6	0.0	18.0	0.0	1.5
Basic retirement benefit	0.4	0.8	0.0	3.4	0.0	0.0
Respondent age	68.4	9.3	45.0	102.0	62.0	75.0
Retirement age	60.4	10.2	28.0	83.0	55.0	67.0
Age first getting Natl. pension	62.4	5.4	47.0	78.0	60.0	64.0
Age first getting basic pension	68.4	5.3	58.0	85.0	65.0	71.0
Self-employed income	0.6	4.5	0.0	115.0	0.0	0.0
Hours per week	4.7	13.8	0.0	84.0	0.0	0.0
Wage	2.1	7.4	0.0	92.0	0.0	0.0

Note: Data are winsorized at the one percent level. The variables with the HH marker are values for households. These moments are calculated after excluding individuals who are self-employed, never-employed, and weakly attached to the labor market.

TABLE A2—RETIREMENT TRANSITION MODELS

	Logit	Probit	Tobit
Fwd. retirement indicator			
Log income	-0.2295***	-0.1328***	-0.0165***
	(0.0019)	(0.0009)	(0.0001)
Log wealth	0.1561***	0.0824***	0.0102***
	(0.0009)	(0.0004)	(0.0001)
Observations	15,492,663	15,492,663	15,492,663
Age FE	Yes	Yes	Yes
Survey Year FE	Yes	Yes	Yes
Health FE	Yes	Yes	Yes
Log income mean (SD)	3.04 (.76)	3.04 (.76)	3.04 (.76)
Log wealth mean (SD)	4.86 (1.6)	4.86 (1.6)	4.86 (1.6)
Log income AME [95% CI]	-.02 [-.02,-.02]	-.02 [-.02,-.02]	-.02 [-.02,-.02]
Log wealth AME [95% CI]	.01 [.01,.01]	.01 [.01,.01]	.01 [.01,.01]

Note: Asymptotic standard errors in parentheses. * p<0.10, ** p<0.05, *** p<0.01. Reported coefficients are untransformed. The reported AME statistics represent the average change in retirement probability associated with a one unit increase in log income or log wealth.

B. *Math Appendix*

Deriving the expected continuation value.

A retiree household with wealth a and lifetime earnings e in period t solves

$$V_t^R(a,e) = \max_{c,a'} u(c) + \beta\left[(1-\delta_R)V_{t+1}^R(a',e) + \delta_R\Omega(a')\right] \quad (16)$$

$$s.t. c + a' \le (1+r_t)a + b_t(e),$$
$$a' \ge 0.$$

where, r_t is the return on assets, $b_t(e)$ is retirement benefit, which depends on lifetime earnings, and Ω reflects the 'warm-glow' utility from bequests.

The problem of a worker household with wealth a, productivity z, and previous earnings e, in period t can be expressed as

$$V_t^W(a,z,e) = \max_{c,l,a'} u(c,\ell) + \mathbf{E}_{\varepsilon_t}\left\{\max_{j=W,R}\left\{\beta\mathbf{E}_{z'|z} V_{t+1}^j\left(a',z',e'\right) - \tau_{Wj} + \nu\varepsilon_t^j\right\}\right\} \quad (17)$$

$$s.t. c + a' \le (1-\tau_t)w_t z\ell + (1+r_t)a,$$
$$e' = g(e, w_t z\ell),$$
$$a' \ge 0.$$

where $\varepsilon \equiv \{\varepsilon_W, \varepsilon_R\}$ is a pair of idiosyncratic preference shocks, which are realized at the end of period t, and τ_{ij} is the cost of switching from life stage i to j.

Preference shocks are distributed according to

$$F(\varepsilon) = \exp(-\exp(-\varepsilon - \bar{\gamma})) \tag{18}$$

where $\bar{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x)) dx$ is Euler's constant and

$$f(\varepsilon) = \partial F / \partial \varepsilon = \exp(-\exp(-\varepsilon - \bar{\gamma})) \exp(-\varepsilon - \bar{\gamma}). \tag{19}$$

We need to solve for the expected continuation value:

$$\Phi_t^i \left(a', e'\right) \equiv \mathbf{E}_{\varepsilon_t} \left[\max_{j=W,R} \left\{ \beta \mathbf{E}_{z'|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \varepsilon_t^j \right\} \right] \tag{20}$$

Choosing life stage j is preferred to k when

$$\beta \mathbf{E}_{z'|z} V_{t+1}^k \left(a', z', e'\right) - \tau_{ik} + \nu \varepsilon_t^k \le \beta \mathbf{E}_{z'|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \varepsilon_t^j \tag{21}$$

$$\Leftrightarrow \nu \varepsilon_t^k \le \beta \left[\mathbf{E}_{z'|z} V_{t+1}^j \left(a', z', e'\right) - \mathbf{E}_{z'|z} V_{t+1}^k \left(a', z', e'\right) \right] - \left(\tau_{ij} - \tau_{ik}\right) + \nu \varepsilon_t^j \tag{22}$$

$$\Leftrightarrow \varepsilon_t^k \le \varepsilon_t^{ijk} \left(a', e'\right) + \varepsilon_t^j \tag{23}$$

where $\bar{\varepsilon}_t^{ijk} \left(a', e'\right) = \frac{1}{\nu} \left\{ \beta \left[\mathbf{E}_{z'|z} V_{t+1}^j \left(a', z', e'\right) - \mathbf{E}_{z'|z} V_{t+1}^k \left(a', z', e'\right) \right] - \left(\tau_{ij} - \tau_{ik}\right) \right\}$ is how much a house-hold values life stage j over k, not including preference shocks.

For a given value ε_t^k, the probability that life stage j is preferred to k is the probability that ε_t^k is small enough:

$$\varepsilon_t^k \le \bar{\varepsilon}_t^{ijk} (a', e') + \varepsilon_t^j \tag{24}$$

which is

$$F\left(\bar{\varepsilon}_t^{ijk} (a', e') + \varepsilon_t^j \right). \tag{25}$$

The expected value of life stage j requires integrating over all possible ε_t^k

$$\int_{-\infty}^{\infty} \left[\beta \mathbf{E}_{z'|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \varepsilon_t^j \right] f\left(\varepsilon_t^j\right) d\varepsilon_t^j \tag{26}$$

and the contribution of j to the value of being in i at t is the value of being in life stage j adjusted for the probability that one would choose j

$$(27)\quad \int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]f\left(\varepsilon_t^j\right)F\left(\bar{\varepsilon}_t^{ijk}\left(a',e'\right)+\varepsilon_t^j\right)d\varepsilon_t^j$$

So the continuation value of someone in life stage j at time t is

$$(28)\quad \Phi_t^i\left(a',e'\right)$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]f\left(\varepsilon_t^j\right)F\left(\bar{\varepsilon}_t^{ij,-j}\left(a',e'\right)+\varepsilon_t^j\right)d\varepsilon_t^j$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}e^{\left(-e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}\right)}e^{\left(-e^{\left(-\bar{\varepsilon}_t^{ij,-j}\left(a',e'\right)-\varepsilon_t^j-\bar{\gamma}\right)}\right)}d\varepsilon_t^j$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}^{j}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}e^{\left(-e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}\left(1+e^{\left(-\bar{\varepsilon}_t^{ij,-j}\left(a',e'\right)\right)}\right)\right)}d\varepsilon_t^j$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}e^{\left(-e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}\left(\sum_{k=W,R}e^{\left(-\bar{\varepsilon}_t^{ijk}\left(a',e'\right)\right)}\right)\right)}d\varepsilon_t^j$$

where the second line follows after replacing $F()$ and $f()$ with (9) and (19), respectively, and the third line uses $\exp(a+b)=\exp(a)\exp(b)$. In the fourth line, we add a summation to go for all life stages (rather than excluding life stage j) because $\bar{\varepsilon}_t^{ijj}(a',e')=0$. Thus, effectively we are replacing the 1 with $exp(0)$.

Defining $\lambda_t^{ij}(a',e')\equiv\log\sum_{k=W,R}\exp(-\bar{\varepsilon}_t^{ijk}(a',e'))$, and considering a change of variables $\zeta_t^j=\varepsilon_t^j+\bar{\gamma}$, we obtain

$$(29)\quad \Phi_t^i\left(a',e'\right)$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\varepsilon_t^j\right]e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}e^{\left(-e^{\left(-\varepsilon_t^j-\bar{\gamma}\right)}\left(\sum_{k=W,R}e^{\left(-\bar{\varepsilon}_t^{ijk}\left(a',e'\right)\right)}\right)\right)}d\varepsilon_t^j$$
$$=\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\left(\zeta_t^j-\bar{\gamma}\right)\right]e^{-\zeta_t^j}e^{-e^{-\zeta_t^j}e^{\lambda_t^{ij}\left(a',e'\right)}}d\zeta_t^j$$

$$(30)\quad =\sum_{j=W,R}\int_{-\infty}^{\infty}\left[\beta\mathbf{E}_{z|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\left(\zeta_t^j-\bar{\gamma}\right)\right]e^{\left(-\zeta_t^j-\exp\left(-\left(\zeta_t^j-\lambda_t^{ij}\left(a',e'\right)\right)\right)\right)}d\zeta_t^j.$$

To go from line 1 to 2, make the substitutions and note that $\partial\zeta^j/\partial\varepsilon^j=1$, so $\partial\zeta^j=\partial\varepsilon^j$. The third line regroups the exponentials using $\exp(a+b)=\exp(a)\exp(b)$.

Now consider another change of variables, $y_t^{ij}=\zeta_t^j-\lambda_t^{ij}$, where we suppress the dependence on (a',e') for ease of notation,

$$\Phi_t^i = \sum_{j=W,R} \int_{-\infty}^{\infty} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\zeta_t^j - \overline{\gamma}\right) \right] e^{\left(-\zeta_t^j - \exp\left(-\left(\zeta_t^j - \lambda_t^{ij}\right)\right)\right)} d\zeta_t^j \tag{31}$$

$$= \sum_{j=W,R} \int_{-\infty}^{\infty} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\tilde{y}_t^{ij} + \lambda_t^{ij} - \overline{\gamma}\right) \right] e^{\left(-\left(\tilde{y}_t^{ij} + \lambda_t^{ij}\right) - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij}. \tag{32}$$

$$= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \int_{-\infty}^{\infty} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\tilde{y}_t^{ij} + \lambda_t^{ij} - \overline{\gamma}\right) \right] e^{\left(-\tilde{y}_t^{ij} - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij}. \tag{33}$$

$$\begin{aligned} &= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\lambda_t^{ij} - \overline{\gamma}\right) \right] \int_{-\infty}^{\infty} e^{\left(-\tilde{y}_t^{ij} - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij} \\ &+ \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \int_{-\infty}^{\infty} \tilde{y}_t^{ij} e^{\left(-\tilde{y}_t^{ij} - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij} \end{aligned}. \tag{34}$$

Now recall that $\int_{-\infty}^{\infty} e^{(-x-\exp(-x))} dx$. Note that $e^{(-x-exp(-x))}$ is the CDF of the standard Gumbel distribution. Also, we note that

$$\int_{-\infty}^{\infty} e^{(-x-\exp(-x))} dx \tag{35}$$

$$= e^{(-x-\exp(-x))} \Big|_{-\infty}^{\infty} \tag{36}$$

$$= 1 - 0 = 1. \tag{37}$$

Hence,

$$\begin{aligned} \Phi_t^i &= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\lambda_t^{ij} - \overline{\gamma}\right) \right] \int_{-\infty}^{\infty} e^{\left(-\tilde{y}_t^{ij} - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij} \\ &+ \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \int_{-\infty}^{\infty} \tilde{y}_t^{ij} e^{\left(-\tilde{y}_t^{ij} - \exp\left(-\tilde{y}_t^{ij}\right)\right)} d\tilde{y}_t^{ij} \end{aligned} \tag{38}$$

$$= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \left(\lambda_t^{ij} - \overline{\gamma}\right) \right] + \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \nu \overline{\gamma} \tag{39}$$

$$= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \lambda_t^{ij} \right] \tag{40}$$

Using the definition $\lambda_t^{ij}(a', e') = \log \sum_{j=W,R} \exp\left(-\overline{\varepsilon}_t^{ijk}(a', e')\right)$, we obtain

$$\begin{aligned} \Phi_t^i\left(a', e'\right) &= \sum_{j=W,R} e^{\left(-\lambda_t^{ij}\left(a', e'\right)\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \lambda_t^{ij}\left(a', e'\right) \right] \\ &= \sum_{j=W,R} e^{\left(-\log \sum_{k=W,R} \exp\left(-\overline{\varepsilon}_t^{ijk}\left(a', e'\right)\right)\right)} \left[\beta \mathbf{E}_{z|z} V_{t+1}^j \left(a', z', e'\right) - \tau_{ij} + \nu \log \sum_{k=W,R} \exp\left(-\overline{\varepsilon}_t^{ijk}\left(a', e'\right)\right) \right] \end{aligned} \tag{41}$$

Substituting the definition of $\overline{\varepsilon}_t^{ijk}\left(a', e'\right) = \dfrac{\beta\left[\mathbf{E}_{z|z} V_{t+1}^j\left(a', z', e'\right) - V_{t+1}^k\left(a', z', e'\right)\right] - \left(\tau_{ij} - \tau_{ik}\right)}{\nu}$, and suppressing the dependence on (a',z',e') for ease of notation, we find that

$$(42)\quad \beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}+\nu\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_{t}^{ijk}\left(a',e'\right)\right)$$

$$(43)\quad =\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}+\nu\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}+\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)$$

$$(44)\quad =\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}+\nu\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)$$

$$(45)\quad =\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}+\nu\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)$$

$$(46)\quad \begin{aligned}&=\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}\\&+\nu\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\right)+\nu\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)\end{aligned}$$

$$(47)\quad =\nu\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)$$

$$(48)\quad =\nu\log\left(\sum_{k=W,R}\exp\left(\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}\right)^{\frac{1}{\nu}}\right)$$

Then

$$(49)\quad \begin{aligned}&\Phi_{t}^{i}\left(a',e'\right)\\&=\sum_{j=W,R}e^{\left(-\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_{t}^{ijk}\left(a',e'\right)\right)\right)}\left[\beta\mathbf{E}_{z'|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}+\nu\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_{t}^{ijk}\left(a',e'\right)\right)\right]\\&=\nu\log\left(\sum_{k=W,R}\exp\left(\beta\mathbf{E}_{z'|z}V_{t+1}^{k}\right)^{\frac{1}{\nu}}\right)\end{aligned}$$

since

$$(50)\quad \sum_{j=W,R}e^{\left(-\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_{t}^{ijk}\left(a',e'\right)\right)\right)}$$

$$(51)\quad =\sum_{j=W,R}e^{\left(-\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}+\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)}$$

$$(52)\qquad =\sum_{j=W,R} e^{\left(-\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)}$$

$$(53)\qquad =\sum_{j=W,R} e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)\right)}$$

$$(54)\qquad =\sum_{j=W,R} e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\right)-\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)\right)}$$

$$(55)\qquad =\sum_{j=W,R} e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\right)\right)} e^{\left(-\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)\right)}$$

$$(56)\qquad = e^{\left(-\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)\right)}\sum_{j=W,R} e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)\right)\right)}$$

$$(57)\qquad =\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{k}-\tau_{ik}}{\nu}\right)\right)^{-1}\sum_{j=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{j}-\tau_{ij}}{\nu}\right)=1$$

Thus, the problem of the worker in (17) can be written as

$$(58)\qquad V_t^W(a,z,e)=\max_{c,l,a'} u(c,\ell)+\nu\log\left\{\sum_{j=W,R}\exp\left(\beta\mathbf{E}_{z'|z}V_{t+1}^{j}\left(a',z',e'\right)-\tau_{ij}\right)^{\frac{1}{\nu}}\right\}$$

$$\begin{aligned} s.t.\ & c+a'\le(1-\tau_t)w_t z\ell+(1+r_t)a,\\ & e'=g(e,\ w_t z\ell),\\ & a'\ge 0.\end{aligned}$$

Deriving the retirement probability.

The probability that a worker retires is given by

$$(59)\qquad \begin{aligned}&\rho_t\left(a',e'\right)\\ &=\Pr\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{R}\left(a',z',e'\right)-\tau_{WR}}{\nu}+\varepsilon_t^R\ge\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^{W}\left(a',z',e'\right)-\tau_{WW}}{\nu}+\varepsilon_t^W\right)\end{aligned}$$

$$(60)\qquad =\int_{-\infty}^{\infty} f\left(\varepsilon_t^R\right)F\left(\bar{\varepsilon}_t^{WRW}\left(a',e'\right)+\varepsilon_t^R\right)d\varepsilon_t^R$$

where, as before, $\bar{\varepsilon}_t^{ijk}\left(a',e'\right)=\frac{1}{\nu}\left\{\beta\left[\mathbf{E}_{z'|z}V_{t+1}^{j}\left(a',z',e'\right)-\mathbf{E}_{z'|z}V_{t+1}^{k}\left(a',z',e'\right)\right]-\left(\tau_{ij}-\tau_{ik}\right)\right\}$ is how much a worker values life stage j over k, scaled by ν. So $F\left(\bar{\varepsilon}_t^{ijk}\left(a',e'\right)+\varepsilon_t^{j}\right)$

is the probability of drawing a ε_t^k smaller than $\bar{\varepsilon}_t^{ijk}\left(a',e'\right)+\varepsilon_t^j$, thus making the household prefer j over k.

Following similar steps as (27)-(39), we know that

$$\rho_t\left(a',e'\right)=\int_{-\infty}^{\infty} f\left(\varepsilon_t^R\right)F\left(\bar{\varepsilon}_t^{WRW}\left(a',e'\right)+\varepsilon_t^R\right)d\varepsilon_t^R \tag{61}$$

$$=\int_{-\infty}^{\infty} e^{\left(-\varepsilon_t^R-\bar{\gamma}\right)} e^{-e^{\left(-\varepsilon_t^R-\bar{\gamma}\right)}\sum_{k=W,R}\left(e^{\left(-\bar{\varepsilon}_t^{WRk}\left(a',e'\right)\right)}\right)} d\varepsilon_t^j \tag{62}$$

$$=e^{\left(-\lambda_t^{WR}\left(a',e'\right)\right)}\int_{-\infty}^{\infty} e^{\left(-\tilde{y}_t^{WR}-\exp\left(-\tilde{y}_t^{WR}\right)\right)} d\tilde{y}_t^{WR} \tag{63}$$

$$=e^{\left(-\lambda_t^{WR}\left(a',e'\right)\right)} \tag{64}$$

Using the definition $\lambda_t^{ij}\left(a',e'\right)\equiv\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_t^{ijk}\left(a',e'\right)\right)$, we obtain

$$\rho_t\left(a',e'\right)=e^{\left(-\lambda_t^{WR}\left(a',e'\right)\right)} \tag{65}$$

$$=e^{\left(-\log\sum_{k=W,R}\exp\left(-\bar{\varepsilon}_t^{WRk}\left(a',e'\right)\right)\right)} \tag{66}$$

$$=e^{\left(-\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}+\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)} \tag{67}$$

$$=e^{\left(-\log\sum_{k=W,R}\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}\right)\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)} \tag{68}$$

$$=e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}\right)\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)\right)} \tag{69}$$

$$=e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}\right)\right)-\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)\right)} \tag{70}$$

$$=e^{\left(-\log\left(\exp\left(-\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}\right)\right)\right)}e^{\left(-\log\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)\right)} \tag{71}$$

$$=\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^R-\tau_{WR}}{\nu}\right)\left(\sum_{k=W,R}\exp\left(\frac{\beta\mathbf{E}_{z'|z}V_{t+1}^k-\tau_{Wk}}{\nu}\right)\right)^{-1} \tag{72}$$

$$=\frac{\exp\left(\beta\mathbf{E}_{z'|z}V_{t+1}^R\left(a',z',e'\right)-\tau_{WR}\right)^{\frac{1}{\nu}}}{\sum_{k=W,R}\exp\left(\beta\mathbf{E}_{z'|z}V_{t+1}^k\left(a',z',e'\right)-\tau_{Wk}\right)^{\frac{1}{\nu}}} \tag{73}$$

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