

Inference and Forecasting Based on the Phillips Curve[†]

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In this paper, we conduct uniform inference of two widely used versions of the Phillips curve, specifically the random-walk Phillips curve and the New-Keynesian Phillips curve (NKPC). For both specifications, we propose a potentially time-varying natural unemployment (NAIRU) to address the uncertainty surrounding the inflation-unemployment trade-off. The inference is conducted through the construction of what is known as the uniform confidence band (UCB). The proposed methodology is then applied to point-ahead inflation forecasting for the Korean economy. This paper finds that the forecasts can benefit from conducting UCB-based inference and that the inference results have important policy implications.

Key Word: Time-varying NAIRU, Random-walk Phillips curve,
New-Keynesian Phillips curve, Uniform confidence
band, Model validation, Inflation forecasting

JEL Code: C12, C13, C14

I. Introduction

Since Milton Friedman introduced the idea of the non-accelerating inflation rate of unemployment (*NAIRU*) in his presidential address to the American Economic Association in 1968, the *NAIRU* has served as a general guideline for those establishing macroeconomic policies. The idea has also been very useful as an empirical basis for predicting changes in the inflation rate. Among the various hypotheses regarding this important structural parameter, there has been a general recognition among many economists that if a *NAIRU* does exist, it must change over time (Stiglitz 1997; Ball and Mankiw 2002).

Perhaps the most widely accepted belief with regard to time-variation in the U.S. *NAIRU* is that the parameter has been falling since the early 1980s (Stiglitz 1997;

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* Received: 2016. 4. 25.

* Referee Process Started: 2016. 4. 26.

* Referee Reports Completed: 2016. 5. 23.

[†] Kim gratefully acknowledges funding from the National Research Foundation of Korea Grant by the Korean Government (NRF-2015S1A5A8014208).

Shimer 1998). Although this consensus on a decreasing *NAIRU* appears reasonable and is supported by empirical research in this area, there remains no formal justification of this hypothesis in the literature. Furthermore, there is a lack of evidence of any time-varying *NAIRU* for economies other than that of the U.S. Given this limitation, the paper conducts inference of the *NAIRU* parameter for the Korean economy and shows how the developed methodology can be applied to forecasting Korean monthly inflation.

To meet these goals, the paper considers two versions of the Phillips curve: the random-walk Phillips curve¹ (Staiger, Stock, and Watson 1996, 1997) and the New-Keynesian Phillips curve (*NKPC*) (Gal'ı and Gertler 1999). Given the uncertainty surrounding the *NAIRU*, we extend the Phillips curves such that the new model framework can deal with uncertainty. That is, the *NAIRU* in the suggested framework does not assume any specific parametric form. Only data determine the unknown form, of which the estimate will be used to forecast inflation.

The unique feature of the approach in this work is that the proposed model validation procedures can be used directly for the forecasting exercise. Thus far, most studies of the Phillips curve in macroeconomics have focused on either model validation or on the forecasting performance, but not both. In this work, we generalize the two widely used versions of Phillips curve such that the modified models can incorporate the uncertainty surrounding the *NAIRU* parameter more efficiently. Uniform inference procedures for the model are proposed and used to suggest a new forecasting methodology that is applied to Phillips-curve-based inflation forecasting. Given that it attempts to combine these two rather distinct areas of research, the current work stands out among the numerous papers related to the forecasting ability of the Phillips curve.

Until recently, the empirical literature on the Phillips curve rarely provided inference based on estimates of the *NAIRU* parameter. A handful of pioneering works in this direction include those by Gordon (1997, 1998) and Staiger, Stock and Watson (1996, 1997, 2001), where the authors estimate a time-varying *NAIRU* in the traditional expectations-augmented Phillips curve (Friedman, 1968; Phelps, 1970) with adaptive expectations and construct its confidence intervals. Unfortunately, these works on the inference of the time-varying *NAIRU* involve only the construction of point-wise confidence intervals of the parameter. In the treatment of dynamic models, such as the *NKPC* with a time-varying *NAIRU*, it is more appropriate and more useful to construct uniform confidence bands (*UCB*) than their point-wise counterparts, as *UCBs* allow us to perform statistical inference for the parameter. That is, the *UCB* allows us to test whether the parameter assumes any specific structure (i.e., constant, linear) on it. In principle, point-wise confidence intervals/bands are not appropriate for testing these hypotheses on the parameter because any inference results based on point-wise outcomes pertain to one specific point only.

In order to construct the asymptotic *UCB* of the time-varying *NAIRU* $U_N(t)$ with the level $100(1-\alpha)\%$, $\alpha \in (0, 1)$ form, it is necessary to find the following

¹The model originates from the expectations-augmented Phillips curve (Friedman 1968; Phelps 1970).

two functions $f_n(\cdot)$ and $g_n(\cdot)$ based on the data:

$$(1) \quad \lim_{n \rightarrow \infty} \mathbb{P}\{f_n(t) \leq U_N(t) \leq g_n(t) \text{ for all } t \in T\} = 1 - \alpha$$

where $T = [0, 1]$. The purpose of constructing the *UCB* above is to test whether the *NAIRU* $U_N(\cdot)$ takes a certain parametric form. That is, using the *UCB* of $U_N(\cdot)$, we are able to test the null hypothesis $H_0: U_N(\cdot) = U_\theta(\cdot)$, where $\theta \in \Theta$ and where Θ is a parameter space. For example, in order to test $H_0: U_\theta(t) = \theta_0 + \theta_1 t$, it is possibly simply to check whether $f_n(t) \leq \hat{\theta}_0 + \hat{\theta}_1 t \leq g_n(t)$ holds for all $t \in T$. Here $\hat{\theta}_0$ and $\hat{\theta}_1$ are the least squares estimates of θ_0 and θ_1 , respectively. If it does hold for all $t \in T$, then we fail to reject the null hypothesis at level α .

In general, the *UCB* is a more conservative confidence band than the traditional point-wise confidence band in the sense that the *UCB* is usually wider than its point-wise counterpart. Thus, test results based on the *UCB* would be more robust than those under the point-wise outcomes. For these reasons, the *UCB* has recently attracted more attention in the econometrics and statistics literature. For example, Baillie and Kim (2015) revisit the forward premium regression approach (Fama 1984) in an effort to understand the potential source of model instability. They undertake the *UCB*-based inference of the model parameters to identify the driving force behind the dynamics and to capture potential incidences of co-movement among the parameters for different currencies. Kim (2016) constructs a *UCB* of the non-parametric trend in a semi-parametric regression model, where the independent variables are non-stationary processes. The *UCB* is then used to test for a parametric specification of the unknown trend in the model. Given these interesting results, we shall construct the *UCB* of a time-varying *NAIRU* and carry out inference about the parameter.

The developed inference procedures can be applied to forecasting inflation variables. Given the *UCB* of *NAIRU*, one can test whether or not a certain parametric form is accepted. If it is accepted, then the estimated structure is used to forecast inflation, involving mainly the extrapolation of the data. If it is rejected, the non-parametric fits of the *NAIRU* can then be used to forecast the variable. Given that non-parametric fits vary over time, we need to combine these time-specific estimates for forecasting. We combine them by averaging the estimates. To compare the performances of the proposed method and of the traditional Phillips curve, the paper conducts a pseudo out-of-sample forecasting experiment. Both the entire sample and sub-samples are utilized to in an assessment of their performances in this experiment. The results and the implications are discussed in detail.

The organization of the paper is as follows: Section II discusses the methodology. The first part is for testing for any potential structural break in the data. We employ a non-parametric break test to rule out any potential bias from specifying a parametric form. The subsequent parts concern the inference that is carried out based on two popular versions of the Phillips curve - the random-walk

Phillips curve (Staiger, Stock, and Watson 1996) and the new Keynesian Phillips curve (Gal'ı and Gertler 1999). The steps used to perform the inference are explained in detail. The section also discusses how forecasting is conducted based on the inference procedures. Essentially, the uniform confidence band is used to select an appropriate model that is eventually used for extrapolation. Section III explains the data used and summarizes and interprets the estimation and forecasting outcomes. This section also discusses the policy implications of the empirical results. Section IV concludes the paper and discusses potential future research. The proof of the theoretic result and the figures and tables are given in the appendix of the paper.

II. Methodology

Inference of the Phillips curve is carried out by the construction of a *uniform* confidence band (*UCB*) (Kim 2015, 2016; Baillie and Kim 2015). The *UCB* is a powerful tool for undertaking the inference of an unknown function in an economic causal model. Unlike the traditional point-wise confidence intervals, the *UCB* can be used for model validation by determining the correct function form. Because this is mostly done by a simple visual check of the result, the entire procedure is also very tractable. As discussed in the introduction, the stability of the *NAIRU* parameter in the Phillips curve is a major source of debate. The *UCB*-based inference method introduced here can be readily used for the validation of the Phillips curve and can potentially contribute to improving the accuracy of inflation forecasts based on the model.

A. Stability of the NAIRU Parameter

One of the issues to consider when applying the Phillips curve to the Korean economy in recent years is the potential parameter instability during the 1997 Asian financial crisis. The possible existence of what is known as a structural break due to this shock can basically invalidate the outcomes of any traditional analysis. Hence, it is desirable to determine this possibility before conducting the *UCB*-based inference of the Phillips curve for the Korean economy. Among the many available tests of change points, we employ a non-parametric subsample-based test (Carlstein 1986) using monthly Korean unemployment data during July of 1982 to May of 2015.² Here, we let U_t denote the monthly unemployment rate at time $t = 1, 2, \dots, n$. We start by partitioning the sample to obtain the following $\{A_i\}$,

$$A_i = \frac{1}{k_n} \sum_{j=1}^{kn} U_{j+ik_n}, \quad i = 0, 1, \dots$$

where $k_n = \lceil n^{2/3} \rceil$. Here $\lceil \cdot \rceil$ is the integer part of a real number. The test statistic

²For the Korean economy, the monthly inflation series starts in July of 1982.

that we utilize is the maximal difference between two adjacent block-wise means:

$$D_n = \max_{1 \leq i \leq m-1} |A_i - A_{i-1}|$$

Here, $m = \lceil n / k_n \rceil$. Under some suitable conditions, one can show that

$$\mathbb{P}\left(\sqrt{\log(m)}\left(k_n^{1/2}\sigma^{-1}D_n - \gamma_m\right) \leq u\right) \Rightarrow \exp\left(-\pi^{-1/2} \exp(-u)\right)$$

where $\gamma_m = [4\log(m) - 2\log(\log(m))]^{1/2}$ and α is the standard deviation of the de-meaned inflation variable. Hence, we reject, at level α , the stability of the *NAIRU* parameter if

$$D_n > D_m := k_n^{-1/2}\sigma\left(\gamma_m + c_\alpha \log(m)^{-1/2}\right)$$

where $c_\alpha = -\log[-\log(1-\alpha)] - 0.5 \log(\pi)$. Here, the standard deviation σ can be estimated with any of the following:

$$\sigma_1 = \frac{\sqrt{\pi k_n}}{2(m-1)} \sum_{i=1}^{m-1} |A_i - A_{i-1}|$$

$$\sigma_2 = \frac{\sqrt{k_n}}{\sqrt{1.348}} \text{median}(|A_i - A_{i-1}|), \quad 1 \leq i \leq m-1$$

$$\sigma_3 = \frac{\sqrt{k_n}}{\sqrt{2(m-1)}} \left(\sum_{i=1}^{m-1} |A_i - A_{i-1}|^2 \right)^{1/2}$$

The test results are reported by Table 1. As shown in the table, the results are mixed depending on which estimate is used to estimate σ . While a break is detected under $\sigma = \sigma_2$, there seems to be not enough evidence of a structural break in the Korean monthly unemployment rate when the other two estimates of σ are used instead, as suitably illustrated by Table 1. Unlike other popular structural break tests that utilize some parametric framework, the test considered here is purely non-parametric in that no model structure is required to carry it out. This may have led to the lack of a consensus among the test results.

TABLE 1—TEST FOR STRUCTURAL BREAK

	D_n	D_m	Result
$\sigma = \sigma_1$	1.9566	2.8475	No break
$\sigma = \sigma_2$	1.9566	1.8791	Break
$\sigma = \sigma_3$	1.9566	3.0244	No break

B. Inference of the Random-Walk Phillips Curve

Given the change-point test results, we undertake the inference of two versions of the Phillips curve: the random-walk Phillips curve and its New Keynesian counterpart. In particular, the random-walk Phillips curve in (2) has been used extensively on the empirical frontier (Staiger, Stock, and Watson 1996, 1997, 2001; Gordon 1997, 1998; Fair 2000; Ball and Mankiw 2002),

$$(2) \quad \pi_i = \pi_{i-1} + \alpha(U_i - U_N) + \epsilon_i,$$

where π_i and U_i represent the inflation and unemployment rates at time $i=1, \dots, n$, respectively. Here, ϵ_i is a zero-mean random error at time i , often dubbed the supply shock, and U_N and α are unknown parameters. The *NAIRU* parameter U_N embeds all shifts in the inflation-unemployment trade-off. The version in (2) employs only one lagged unemployment value, while multiple lag terms can be introduced without changing the methodology. For simplicity, we use one lag term.

In principle, U_N can exhibit substantial variation over time (Gordon 1997, 1998; Staiger, Stock, and Watson 1996, 1997; Ball and Mankiw 2002). To address this possibility, we build on the traditional random-walk-type model (2) and propose the following Phillips curve with the potentially time-varying *NAIRU* $U_N(\cdot)$,

$$(3) \quad \pi_i = \pi_{i-1} + \alpha \left[U_i - U_N \left(\frac{i}{n} \right) \right] + \epsilon_i$$

where $U_N(\cdot)$ varies over time in its domain $[0, 1]$. The time-varying *NAIRU* $U_N(t)$ can be estimated by the semi-parametric two-step estimator proposed by Kim (2016),

$$(4) \quad \tilde{U}_N(t) = \sum_{i=1}^n w_n(t, i) \left(U_i - \frac{\Delta \pi_i}{\hat{\alpha}_D} \right)$$

where $\Delta \pi_i = \pi_i - \pi_{i-1}$, and $\hat{\alpha}_D$ is the differencing estimate of the fixed parameter α in (3). Here, $w_n(t, i) = K \left(\frac{t-i/n}{b_n} \right) \frac{S_2(t) - (t-i/n)S_1(t)}{S_2(t)S_0(t) - S_1^2(t)}$ with

$$S_j(t) = \sum_{i=1}^n (t-i/n)^j K \left(\frac{t-i/n}{b_n} \right), \quad j=0,1,2. \quad \text{For the kernel function } K(\cdot), \text{ we}$$

employ the Epanechnikov kernel $K(x) = 3 \max(1-x^2, 0) / 4$. The bandwidth b_n is established by the generalized cross-validation (*GCV*) method (Craven and

Wahba 1979). The main idea in (4) is that we first estimate α by a first-differencing approach (Yatchew 1997) and then employ a local-smoothing technique based on the first-differencing estimate to estimate the unknown trend. We refer to Kim (2016) for details. According to both (4) and Theorem 2 in Kim (2016), the UCB of $U_N(\cdot)$ in (3) is constructed as follows:

- (i) Select the bandwidth b_n by means of generalized cross-validation (GCV) (Craven and Wahba 1979) and obtain the two-stage semi-parametric estimate (Kim 2016) under the Epanechnikov kernel. To deal with the under-smoothing issue, one can consider a *bias-corrected* estimator instead.
- (ii) Compute $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right|$, where the $\{Z_i\}$ values are generated IID standard normal random variables.
- (iii) Repeat (ii), for instance 1,000 times. We obtain the 95th quantile of the sampling distribution of $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right|$, and denote it as $\hat{q}_{0.95}$.
- (iv) Estimate σ_ϵ using the following variant of the subseries variance estimator proposed by Carlstein (1986):

$$(5) \quad \hat{\sigma}_\epsilon^2 = \frac{1}{2(m-1)k_n} \sum_{i=1}^{m-1} \left[\sum_{j=1}^{k_n} \left(U_{j+ik_n} - U_{j+(i-1)k_n} - \left(\Delta\pi_{j+ik_n} - \Delta\pi_{j+(i-1)k_n} \right) / \hat{\alpha}_D \right) \right]^2$$

where k_n is the length of the subseries and $m = \lceil n / k_n \rceil$ is the largest integer not exceeding n / k_n . Carlstein (1986) shows that the optimal length of the subseries is $k_n \asymp n^{1/3}$. Hence, we let $k_n \asymp n^{1/3}$ here. For a finite sample, we choose $k_n = \lceil n^{1/3} \rceil$. The asymptotic consistency of $\hat{\sigma}_\epsilon^2$ with regard to the long-run variance σ_ϵ^2 is given by Lemma 5 in Kim (2016).

- (v) According to Theorem 2 in Kim (2016), the 95% UCB of $U_N(t)$ is $[\tilde{U}_N(t) \pm \hat{\sigma}_\epsilon \hat{q}_{0.95}]$.

Note here that the above UCB is more effective in a finite sample than the usual UCB based on the asymptotic results, as that suggested here avoids the problem of slow convergence. For more on this issue, we refer to Theorem 2 and the following discussion in Kim (2016).

C. Inference of the New Keynesian Phillips Curve (NKPC)

For the $NKPC$ side, we consider the hybrid $NKPC$ (Gal'1 and Gertler 1999) based on the unemployment gap. Although it is a theoretically coherent framework,

the original *NKPC* is known to have several empirical limitations, including that related to its ability to forecast inflation. This led to the development of the following hybrid *NKPC* framework,

$$\pi_i = \delta(U_i - U_N) + (1 - \varphi)\mathbb{E}(\pi_{i+1} | \mathcal{I}_i) + \varphi\pi_{i-1}$$

where π_i and U_i are inflation and unemployment rate at time $i = 1, \dots, n$, and U_N is the *NAIRU* parameter. Here $0 < \varphi < 1$ controls the inflation persistence and δ is related to the parameter that governs the degree of price stickiness. The *NKPC* proposed in this work is the unemployment-gap-based hybrid *NKPC* with a time-varying *NAIRU* $U_N(\cdot)$:

$$(6) \quad \pi_i = \delta(U_i - U_N(i/n)) + (1 - \varphi)\mathbb{E}(\pi_{i+1} | \mathcal{I}_i) + \varphi\pi_{i-1}$$

For the inflation series π_i in (6), we assume that

$$(7) \quad \pi_i = \mu_\pi(i/n) + \pi_i^d,$$

where the unknown mean $\mu_\pi(\cdot)$ of inflation is Lipschitz-continuous over $[0, 1]$, and the demeaned inflation π_i^d is a mean-zero stationary random process such that

$$(8) \quad \pi_i^d = \mathcal{F}(\mathcal{I}_i),$$

where $\mathcal{F}(\cdot)$ is a measurable function and $\mathcal{I}_i = (\dots, \epsilon_{i-2}, \epsilon_{i-1}, \epsilon_i)$ is the information set available up to time i . The framework in (8) is general such that both linear and non-linear time series processes such as the ARCH process (Engle 1982) can be represented in this way. Moreover, the de-meaned inflation $\{\pi_i^d\}$ is assumed to have a finite fourth moment. Under (7) and (8), the hybrid *NKPC* in (6) can be written as follows,

$$(9) \quad \delta(U_i - U_N(i/n)) = \mu_\pi^*(i/n) + \pi_i^*$$

where $\mu_\pi^*(i/n)$ and π_i^* are defined respectively by

$$(10) \quad \mu_\pi^*\left(\frac{i}{n}\right) := \mu_\pi\left(\frac{i}{n}\right) - (1 - \varphi)\mu_\pi\left(\frac{i+1}{n}\right) - \varphi\mu_\pi\left(\frac{i-1}{n}\right)$$

and

$$(11) \quad \pi_i^* := \pi_i^d - \varphi\pi_{i-1}^d - (1 - \varphi)\mathbb{E}(\pi_{i+1}^d | \mathcal{I}_i),$$

According to (7) and (8), π_i^* is a mean-zero stationary random process. Moreover, because $\mu_\pi(\cdot)$ is Lipschitz-continuous over $[0, 1]$, one can show that

$$(12) \quad \left| \mu_\pi^* \left(\frac{i}{n} \right) \right| \leq \left| \mu_\pi \left(\frac{i+1}{n} \right) - \mu_\pi \left(\frac{i}{n} \right) \right| + \varphi \left| \mu_\pi \left(\frac{i+1}{n} \right) - \mu_\pi \left(\frac{i-1}{n} \right) \right| = O \left(\frac{1}{n} \right)$$

Thus, by applying (12) to (9), we can rewrite the hybrid *NKPC* with a time-varying *NAIRU* in (6) as in the following equation:

$$(13) \quad U_i = U_N \left(\frac{i}{n} \right) + O \left(\frac{i}{n} \right) + e_i$$

Here, $e_i = \pi_i^* / \delta$ is mean-zero stationary due to the mean-zero stationarity of π_i^* . In addition, the inflation variables in (6) are included as $O(1/n)$ and e_i . Equation (13), derived from the model (6), will serve as the main workhorse in estimating and constructing the *UCB* of the *NAIRU* parameter in the hybrid *NKPC* here. Given equation (13), we propose a local-linear regression (Cleveland 1979) estimate of the time-varying *NAIRU* $U_N(\cdot)$ because this method minimizes the well-known boundary problem in the kernel-based regression process. The estimation of $U_N(\cdot)$ can be done by the following local-linear regression,

$$(14) \quad \hat{U}_N(t) = \sum_{i=1}^n w_n(t, i) U_i$$

Where $w_n(t, i) = K \left(\frac{t-i/n}{b_n} \right) \frac{S_2(t) - (t-i/n)S_1(t)}{S_2(t)S_0(t) - S_1^2(t)}$ with

$S_j(t) = \sum_{i=1}^n (t-i/n)^j K \left(\frac{t-i/n}{bn} \right)$. As in (4), the Epanechnikov kernel is used and the bandwidth is chosen by GCV. The time domain of t is fixed over $t \in [0, 1]$ and $w_n(t, i)$ is the weight given to each observation. The asymptotic consistency of the local-linear estimate $\hat{U}_N(t)$ is provided by Kim (2016).

To carry out inference of the *NAIRU* in (6), one can employ the idea of uniform inference as in the previous section. Given its estimate in (14), the uniform confidence band (*UCB*) of $U_N(\cdot)$ in (6) can be constructed. The theoretic justification of the methodology is provided by the following:

Theorem 1. (Invariance Principle) Let $\hat{U}_N(t)$ be the estimator from (14). According to $nb_n^3 + 1 / (nb_n^2) = o(1)$ and given the trend-stationarity in (7) and (8),

$$(15) \quad \sqrt{\frac{nb_n}{\log(n)}} \sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} - \sum_{i=1}^n w_n(t, i) Z_i \right| = o_{\mathbb{P}}(1)$$

where $\sigma_e^2 = \sum_{k \in \mathbb{Z}} \mathbb{E}(e_0 e_k)$ is the long-run variance of $e_i = \pi_i^* / \delta$ in (13). Here, Z_i is an IID standard normal random variable.

The main idea in the proof of (15) is provided in the Appendix. The invariance principle in Theorem 1 states that we can approximate the quantiles of

$\sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} \right|$ using the quantiles of the sampling distribution of $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right|$, because we have $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right| = O_{\mathbb{P}} \left(\sqrt{\frac{\log(n)}{nb_n}} \right)$ from Kim (2016). This is an important and useful result because it

means that we can easily approximate the quantiles of the proposed test statistic using IID standard normal random variables instead. Without this result, we have to

use the asymptotic distribution of $\sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} \right|$ in order to construct

the uniform confidence bands of $U_N(t)$. However, this approach should be used

with great caution because the asymptotic distribution $\sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} \right|$ is

an extreme-value (or Gumbel) distribution (Kim 2015). It is well known that convergence to this distribution is extremely slow and that the confidence bands based directly on this distribution could be very inaccurate if the sample size is not large enough. Given Theorem 1, we propose the following steps to carry out the uniform inference of *NAIRU*:

- (i) Select the optimal bandwidth bn for our local-linear regression (14) based on the generalized cross-validation (*GCV*) method (Craven and Wahba 1979).
- (ii) Obtain the local-linear estimate $\hat{U}_N(t)$ proposed in (14). Here, we use an Epanechnikov kernel.
- (iii) Compute $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right|$, where $w_n(t, i)$ is the weight for local-linear regression in (14), and the $\{Z_i\}$ values are generated IID standard normals.
- (iv) Repeat (iii), for instance 1,000 times. We obtain the 95th quantile of this sampling $\sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) Z_i \right|$, and denote it as $\hat{q}_{0.95}$.

- (v) Estimate σ_e using the following *subseries variance estimator* proposed by Carlstein (1986) and extended by Kim (2016),

$$(16) \quad \hat{\sigma}_e^2 := \frac{1}{2(m-1)k_n} \sum_{i=1}^{m-1} \left[\sum_{j=1}^{k_n} (U_{j+ik_n} - U_{j+(i-1)k_n}) \right]^2$$

where k_n is the length of the subseries and $m = \lceil n / k_n \rceil$ is the largest integer not exceeding n / k_n . Carlstein (1986) shows that the optimal length of the subseries is $kn \asymp n^{1/3}$. In practice, we choose $k_n \in (n^{1/3}, n^{1/2})$. The asymptotic consistency of $\hat{\sigma}_e^2$ to σ_e^2 is given by Carlstein (1986) and Kim (2016).

- (vi) The 95% *UCB* of $U_N(t)$ is $[\hat{U}_N(t) \pm \hat{\sigma}_e \hat{q}_{0.95}]$.

As in the case of the random-walk Phillips curve, the above *UCB* is more effective for inference with a finite sample than the usual *UCB* based on asymptotic results because the proposed method allows us to avoid the problem of slow convergence. The constructed *UCB* will be used to test various hypotheses regarding the *NAIRU*, such as the hypothesis that it has been falling since the early 1980s. A detailed description of the data and the empirical results will be provided in the following section.

D. Inflation Forecasting

One of the main purposes of using the Phillips curve in practice is to forecast inflation series. Given the uniform inference procedures developed here, one carry out the forecasting through model validation. If a parametric model is justified through uniform inference, then the model is then used to generate forecasts. If not, alternative semi-parametric fits can be used to forecast the variable. That is, the *UCB* can provide the model selection criterion for forecasting. Specifically, we propose the following steps to forecast monthly inflation for the Korean economy:

- (i) Test for a structural break.
- (ii) If there is a break, then reduce the sample to the post-break period. Otherwise, use the entire sample.
- (iii) Construct the uniform confidence band (*UCB*) of the *NAIRU* parameter.
- (iv) Test the null hypothesis of a constant *NAIRU* based on the constructed *UCB*.
- (v) If the null hypothesis is accepted, inflation is forecast based on the estimate of the constant *NAIRU*.
- (vi) If not, use the average of the non-parametric estimates for the *NAIRU* to forecast inflation.

In this experiment, we generate point-ahead forecasts of inflation and obtain the forecast errors. A subset of the sample is used to estimate the Phillips curve. Given the estimate, the underlying model is updated to the next time point. Using the first-stage estimate and the updated covariate, the inflation variable is forecast. In principle, the above procedures are applied to the sample after the potential break date only. However, we also apply them to the entire sample for reference such that the forecasting results based on the two samples can be compared.

III. Empirical Results

The data are obtained from the homepage at the Bank of Korea (<http://www.bok.or.kr>). They include the monthly consumer price index (CPI) and monthly unemployment rate from July of 1982 to May of 2015. The CPI is converted to the monthly inflation rate before it is used with the two Phillips curves. Given the potential break at the end of 1997, the sample is divided into the pre-break period (until the end of 1997) and the post-break period (the remaining sample). In the forecast of inflation, we employ both the entire sample and the sample of the post-break period only. Regarding the inference on the *NAIRU* parameter, both the random-walk Phillips curve and the new Keynesian Phillips curve (*NKPC*) are used. First, the inference results for the random-walk Phillips curve and for the *NKPC* are summarized in Figures 1 and 2, respectively.

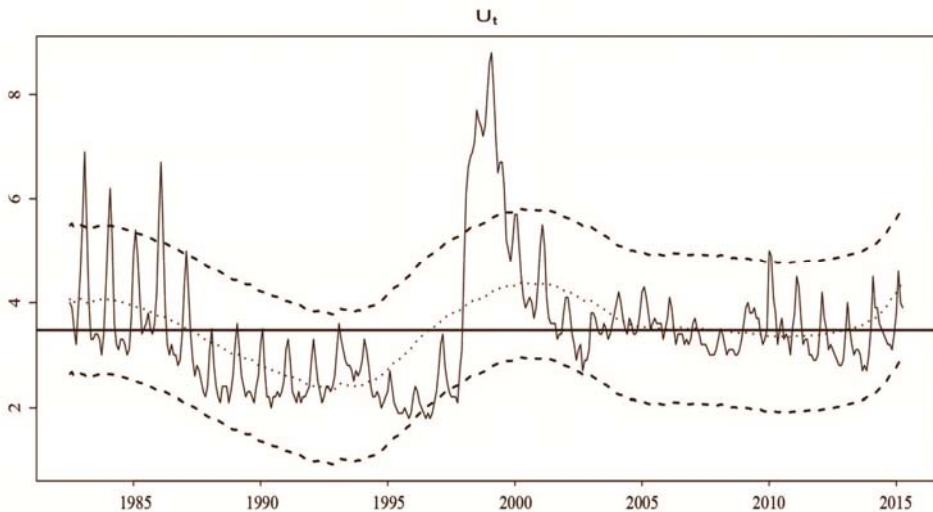


FIGURE 1. TIME-VARYING *NAIRU* FOR THE KOREAN ECONOMY (JULY 1982 – MAY 2015)

Note: The curve (dotted) in the middle of the band is a local-linear estimate of *NAIRU* in the **Random-Walk Phillips Curve**. For the local-linear regression, we use an Epanechnikov kernel. The GCV chooses $bn = 0.15$. The band (dashed) is 95% uniform confidence band (*UCB*) of *NAIRU*. The estimate of *NAIRU* and the *UCB* are placed over the monthly unemployment rates (light solid). The fitted horizontal line for a constant *NAIRU* (dark solid) is $U = 3.47$.

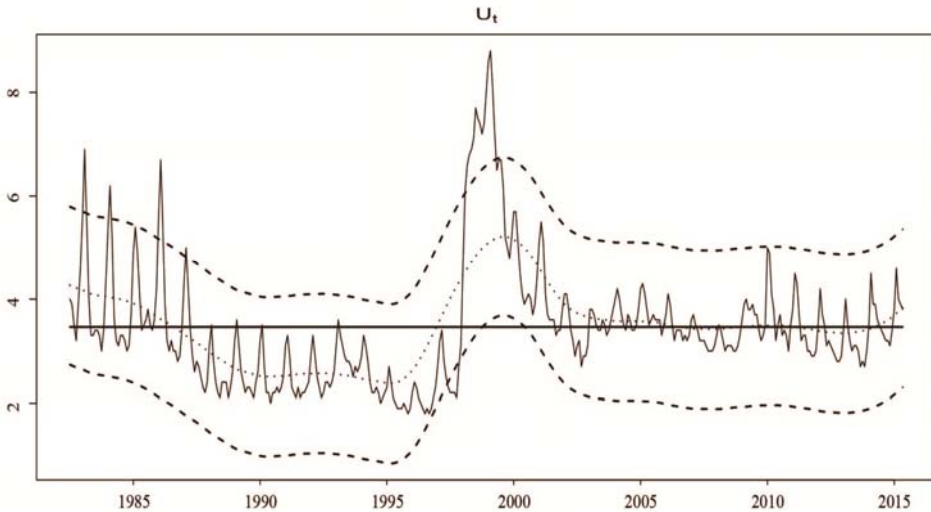


FIGURE 2. TIME-VARYING *NAIRU* FOR THE KOREAN ECONOMY (JULY 1982 - MAY 2015)

Note: The curve (dotted) in the middle of the band is a local-linear estimate of the *NAIRU* in the **New Keynesian Phillips Curve**. For local-linear regression, we use an Epanechnikov kernel. The GCV chooses $bn = 0.09$. The band (dashed) is the 95% uniform confidence band (*UCB*) of the *NAIRU*. The estimates of the *NAIRU* and the *UCB* are shown over the monthly unemployment rates (light solid). The fitted horizontal line for a constant *NAIRU* (dark solid) is $U = 3.46$.

A. Inference of the *NAIRU*

Figure 1 reports the monthly unemployment data (light solid) and the estimate of the fixed *NAIRU* (dark solid) in the random-walk Phillips curve. The estimate of the traditional fixed *NAIRU* for July of 1982 to May of 2015 is 3.47(%). The light-dotted curve is the semi-parametric fit in (4), and the surrounding band (dark-dotted) is the 95% uniform confidence band (*UCB*) of the *NAIRU* parameter. As shown in the figure, the semi-parametric fits clearly show the time variation of the *NAIRU* during this period. The *NAIRU* decreases during the economic expansion of the late 1980s and the early 1990s. The estimate rises in the late 1990s and reaches its peak just after the 1997 financial crisis. Then, it starts declining again and remains around the fixed estimate from that point. The variation in the cyclical unemployment matches the Korean business cycles during the period, which indicates that the model-based semi-parametric *NAIRU* estimates are reasonable.

One of the advantages of using the results in Figure 1 is that they enable the uniform inference of the *NAIRU*. In order to accept a certain null hypothesis for the *NAIRU* parameter, the null value must be contained by the *UCB* over the entire period. Otherwise, the null hypothesis is rejected. For example, if the 95% *UCB* contains the estimate of the fixed *NAIRU* during the period of July of 1982 to May of 2015, the hypothesis of a constant *NAIRU* during the period is accepted. If not, the null is rejected at the 5% level. The important point is that the null value must be contained by the *UCB* during the entire period to be accepted. Figure 1 indicates that the hypothesis of a constant *NAIRU* is accepted at the 5% level because the fixed estimate is entirely contained within the 95% *UCB*.

In contrast, Figure 2 illustrates the estimation and inference results under the

NKPC during the same period. The estimate of a fixed *NAIRU* under *NKPC* is shown by the horizontal line at 3.46%. That is, there is little difference in the fixed *NAIRU* estimate between the random-walk Phillips curve and the *NKPC*. As before, the non-parametric estimate of the time-varying *NAIRU* and its 95% *UCB* for the *NKPC* are shown by the light-dotted and dark-dotted curves, respectively. Although they agree in general, the finer results under the *NKPC* and those based on the random-walk Phillips curve differ. For example, the maximum value of the time-varying *NAIRU* under the *NKPC* is higher than that in the random-walk case. However, the increase in the *NAIRU* at the end of the sample period is higher under the random-walk Phillips curve than in the *NKPC* case. Otherwise, the results under the different models are in general agreement.

The most noticeable difference between Figure 1 and Figure 2 is that the hypothesis of a constant *NAIRU* is rejected at the 5% level for the *NKPC*. The 95% *UCB* presented in Figure 2 fails to contain the horizontal estimate of a fixed *NAIRU* at the turn of the century. The dramatic increase in the *NAIRU* from the late 1990s raises the confidence level as well, and this increase eventually leads to the rejection of the null hypothesis at the 5% level. As noted above, the relatively high increase in the *NAIRU* estimate under the *NKPC* during the late 1990s explains why there is a change in the test result. Indeed, Figures 1 and 2 provide useful information regarding the potential variation in the *NAIRU*. However, they also raise the issue of robustness given the range of possible models to consider in the inference process.

B. Inflation Forecasting

For forecasting inflation based on the Phillips curves, we employ both the full sample (July of 1982 to May of 2015) and the post-break sample (January of 1998 to May of 2015). For each sample, a pseudo-out-of-sample forecasting experiment is conducted based on the random-walk Phillips curve using the first half of the observations.

In each case, both a rolling window of a fixed length and an expanding window with an increasing length are utilized to assess the robustness of the results. To measure the accuracy of point-ahead inflation forecasts, the standard mean-absolute-error (MAE) and the root-mean-squared-error (RMSE) are used. The forecasting results are summarized in Tables 2 and 3. In both tables, “fixed *NAIRU*” refers to the forecast results under the constant *NAIRU*, while “time-varying *NAIRU*” means that the forecasting is carried out through validation of the *UCB*-based model.

Table 2 shows the forecast results based on the full sample. Each error is divided by the lowest corresponding error. For example, the RMSE under the fixed *NAIRU* when the expanding window is used is divided by the RMSE under the time-varying *NAIRU* because the latter is smaller than the former, and so forth. In each case, the error measure under the time-varying *NAIRU* is lower than that under the fixed *NAIRU*, indicating that the inflation forecasts obtained through the inference procedure are more accurate than those based on the fixed *NAIRU* estimate. As shown in Table 3, the same pattern carries over to the case when only the post-break data are used to forecast the monthly inflation. Although the forecast gain is

TABLE 2—FORECAST ERRORS: FULL SAMPLE
(EACH ERROR IS DIVIDED BY THE LOWER CORRESPONDING ERROR)

	expanding window		rolling window	
	Fixed <i>NAIRU</i>	Time-varying <i>NAIRU</i>	Fixed <i>NAIRU</i>	Time-varying <i>NAIRU</i>
RMSE	1.1104	1.0000	1.1441	1.0000
MAE	1.0807	1.0000	1.1024	1.0000

TABLE 3—FORECAST ERRORS: POST-BREAK SAMPLE
(EACH ERROR IS DIVIDED BY THE LOWER CORRESPONDING ERROR)

	expanding window		rolling window	
	Fixed <i>NAIRU</i>	Time-varying <i>NAIRU</i>	Fixed <i>NAIRU</i>	Time-varying <i>NAIRU</i>
RMSE	1.0918	1.0000	1.0779	1.0000
MAE	1.0814	1.0000	1.0889	1.0000

not great in both cases, the results in Tables 2 and 3 confirm that one can clearly benefit from generating inflation forecasts through the *UCB*-based inference approach suggested in this study.

C. Policy Implications

In macroeconomics, the *NAIRU* parameter plays an important role because this structural parameter allows us to determine the current status of the economy in the business cycle. If the current unemployment rate is below the *NAIRU*, the economy is believed to be undergoing an economic expansion. Otherwise, it is in recession. Given an alternative means of estimating and conducting inference on this important parameter, we discuss the policy implications of the empirical results here.

Figures 1 and 2 demonstrate that there are multiple time points during which the unemployment rate is located between the fixed *NAIRU* estimate and the smoothly varying semi-parametric estimate. From Figure 1, the Korean unemployment rate is above the semi-parametric estimate and below the fixed estimate during much of the first half of the 1990s. According to the fixed *NAIRU*, the Korean economy expanded during this time. However, the semi-parametric estimate says the opposite: the economy went through a recession. This finding has significant policy implications due to they need to introduce completely different policy changes depending on which estimate to believe. If using the fixed estimate of 3.47%, it becomes necessary to stabilize the economy with certain contractionary policies. If policymakers use the semi-parametric measure instead, they need to stimulate the economy by introducing expansionary policies.

Given these two completely different options on the table, policymakers may want to resort to using the *UCB*-based inference of the Phillips curve. That is, if the constructed *UCB* accepts the hypothesis of a constant *NAIRU* by completely covering it, it may be wise to determine that the economy is undergoing an expansion and to change policies accordingly. In contrast, if the *UCB* rejects the hypothesis, it would be reasonable to believe that the economy is in a recession and

to stimulate it by introducing the proper policies. Given the gravity of the consequence of adopting incorrect policies, it is crucial to decide wisely when determining the status of the business cycle. Clearly, the *UCB*-based inference methodology in this work can be used to achieve this goal.

Regarding this issue, one also must consider the robustness of the inference. The inference results based on the two models here are quite contradictory: the *UCB* based on the random-walk Phillips curve accepts the fixed *NAIRU* hypothesis, whereas that based on the *NKPC* rejects it. That is, one cannot use the fixed *NAIRU* estimate for policy analysis if the *NKPC* is believed to be the underlying model. Given that we understand how different the policy suggestion could be depending on which *NAIRU* estimate to trust, it is important to be able to determine which model is the true underlying framework. Unfortunately, the methodology developed in this work applies only to the selection of the correct form for the *NAIRU* parameter. Further research is needed to develop a methodology to determine the proper model framework.

D. Comparison to Business Cycle Measures

Given the potential difference between business cycle decisions based on the traditional fixed *NAIRU* and the time-varying case, we can compare the results under these two different specifications and the official business cycle decisions announced by the South Korean Government on a routine basis. Among the most standard measures of the business cycle are those by the Korean Statistical Information Service (KOSIS), which are announced every month. The data contain binary values: either zero (i.e., a recession) or one (i.e., an expansion). The sample is trimmed for a comparison between the official decisions by KOSIS and the decisions under our methodology during the period of July of 1982 to July of 2011. The frequency of the data is monthly, which gives us a total of 349 decisions.

We first compare the KOSIS decisions on the Korean business cycle and those made with the time-varying *NAIRU* estimates in order to observe the percentage of these decisions that matches. Because the original monthly Korean unemployment data are very irregular, we perform some preliminary smoothing before comparing them to the time-varying *NAIRU* estimates. The data shows that approximately 71 percent of the business cycle decisions during the period of July of 1982 to July of 2011 match, with the number slightly increasing to approximately 73 percent between the fixed-*NAIRU*-based decisions and the KOSIS announcements.

Two features are noteworthy in this outcome. First, the majority of these decisions based on the different approaches appear to agree in general, although there is also quite a considerable amount of discrepancy among the decisions. In a sense, some degree of difference among the results is predictable because the three methodologies are fundamentally different. Second, the decisions based on the fixed *NAIRU* hypothesis appear to be marginally closer to the KOSIS decisions than those based on the time-varying *NAIRU*. One potential reason for this outcome stems from the methodology used for the KOSIS decisions, which is likely to be based on the traditional hypothesis of a “fixed” *NAIRU*, although we are not entirely sure of the particular methodology employed for the decision. Because we innovate with the traditional fixed-*NAIRU* assumption in this paper,

the results derived under the proposed methodology are likely differ from the decisions based on the traditional assumption regarding the parameter, which is what we observe in this experiment. The reported difference among the business cycle decisions makes it very important to have some reliable inference procedures for the potentially time-varying *NAIRU* parameter. The methodology proposed in this work can be used to such an end.

IV. Conclusion

In this study, we consider two widely used version of the Phillips curve: the random-walk Phillips curve (Staiger, Stock and Watson 1996, 1997) and the New-Keynesian Phillips curve (*NKPC*) (Gal'ı, J. and Gertler 1999). We undertake uniform inference of each model and check whether the empirical data support the representative parametric framework. The inference is conducted through the construction of a uniform confidence band (*UCB*) for the *NAIRU*. It was found that the widely believed constancy of the *NAIRU* is rejected under the *NKPC* for the Korean economy, whereas parameter constancy is accepted under its random-walk counterpart.

We apply the developed methodology to inflation forecasting. If the parametric fit is entirely covered by the constructed *UCB*, the in-sample fit is extended out-of-sample to forecast the inflation variable. If the fit is not covered by the *UCB*, we resort to the averaged semi-parametric fits of the time-varying *NAIRU*. In a pseudo-out-of-sample forecasting experiment conducted here, the forecasts under this method and those under the traditional random-walk Phillips curve are compared to assess their relative advantages. For both the entire sample and the post-break sample, the *UCB*-based forecasts are found to be superior to those based on the traditional approach. The superiority of the *UCB*-based forecasts persists under both the rolling-window and expanding-window schemes.

The current project leaves a number of interesting topics for potential future research. First, the paper uses the uniform inference methodology to choose between the fixed *NAIRU* estimate and the smoothly time-varying cases for inflation forecasting. In fact, the presence of structural breaks is highly likely in the Korean economy. At the same time, both the number of potential breaks and their dates are uncertain. To address this uncertainty, one can perform forecast averaging based on the *UCB*. Assuming that any date in the sample could be a potential break date, we forecast based on parametric models with breaks that are justified by the *UCB* only. The forecasts accepted by the *UCB* are then combined through averaging. Because this approach handles model uncertainty via an averaging method, the accuracy of the forecasts could be higher than that of the forecasts here.

Another interesting extension would be to develop a methodology for selecting the correct Phillips curve in the beginning. The current work assumes that the correct Phillips curve to use is either the random-walk curve or the *NKPC*. Inference is then conducted on the model parameter. In this sense, the approach here is semi-parametric. However, no justification of the assumption is provided in the current work. As shown in Figures 1 and 2, the inference outcome could be

rather model-sensitive, and it lacks in robustness. By providing some reliable guideline on the issue, we can make the current result more robust and reliable from the perspective of policy analysis. Further insight can be gained by extending the work in these and in other directions.

APPENDIX

The proof of Theorem 1 is based on the trend-stationarity of the inflation variable and the invariance principle in Kim (2015). We provide here only a sketch of the proof. For details of the proof, we refer the reader to Kim (2015).

Proof of Theorem 1

Recall that we have the hybrid *NKPC* in equation (9) due to (7) and (8). By performing a Taylor's expansion on $U_N(\cdot)$ in (9), we can show the following,

$$\begin{aligned}
 & \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) [U_i - U_N(t)] - \sigma_e \sum_{i=1}^n w_n(t, i) Z_i \right| \\
 &= \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) [U_N(i/n) + O(1/n) + e_i - U_N(t)] - \sigma_e \sum_{i=1}^n w_n(t, i) Z_i \right| \\
 &= \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) [U_N(i/n) - U_N(t)] + O(1/n) + \sum_{i=1}^n w_n(t, i) (e_i - \sigma_e Z_i) \right| \\
 (17) \quad &= \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t-i/n) w_n(t, i) U'_N(t) \right| + C \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n (t-i/n)^2 w_n(t, i) U''_N(t) \right| \\
 &\quad + O(1/n) + \sup_{0 \leq t \leq 1} \left| \sum_{i=1}^n w_n(t, i) (e_i - \sigma_e Z_i) \right| \\
 &= O(b_n) + O(b_n^2) + O(1/n) + o_{\mathbb{A}\mathbb{S}} \left(\frac{\log(n)}{n^{3/4} b_n} \right)
 \end{aligned}$$

where C is some constant. The last equality is due to the smoothness of the *NAIRU*, the Lipschitz-continuity of the kernel, and Lemma 2 in Kim (2015). Then, according to $\sum_{i=1}^n w_n(t, i) = 1$, we have

$$\sqrt{nb_n} \sup_{0 \leq t \leq 1} \left| \hat{u}^*(t) - u^*(t) - \sigma_e \sum_{i=1}^n w_n(t, i) Z_i \right| = O(n^{1/2} b_n^{3/2}) + O\left(\sqrt{\frac{bn}{n}}\right) + o_{\mathbb{A}\mathbb{S}} \left(\frac{\log(n)}{n^{1/4} b_n^{1/2}} \right)$$

Then, with $nb_n^3 + 1/(nb_n^2) = o(1)$,

$$\sqrt{nb_n} \sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} - \sum_{i=1}^n w_n(t, i) Z_i \right| = o_{\mathbb{P}}(1)$$

which leads to

$$\sqrt{\frac{nb_n}{\log(n)}} \sup_{0 \leq t \leq 1} \left| \frac{\hat{U}_N(t) - U_N(t)}{\sigma_e} - \sum_{i=1}^n w_n(t, i) Z_i \right| = o_{\mathbb{P}}(1)$$

□

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